MAD-NG Reference Manual (DRAFT)

Methodical Accelerator Design
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Abstract
The Methodical Accelerator Design – Next Generation application is an all-in-one standalone versatile tool for particle accelerator design, modeling, and optimization, and for beam dynamics and optics studies. Its general purpose scripting language is based on the simple yet powerful Lua programming language (with a few extensions) and embeds the state-of-art Just-In-Time compiler LuaJIT. Its physics is based on symplectic integration of differential maps made out of GTPSA (Generalized Truncated Power Series). The physics of the transport maps and the normal form analysis were both strongly inspired by the PTC/FPP library from E. Forest. MAD-NG development started in 2016 by the author as a side project of MAD-X, hence MAD-X users should quickly become familiar with its ecosystem, e.g. lattices definition.

`http://cern.ch/mad`

Keywords
Methodical Accelerator Design; Accelerator beam physics; Scientific computing; JIT compiler; C and Lua programming.

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Foreword

This textbook is in a very draft version...

Temporary abbreviations used in the text:

`TBD`	To Be Documented
`TBR`	To Be Reviewed
`NYI`	Not Yet Implemented
`Todo`	To Do

Part I

General

Chapter 1. Introduction

1 Presentation

The Methodical Accelerator Design – Next Generation application is an all-in-one standalone versatile tool for particle accelerator design, modeling, and optimization, and for beam dynamics and optics studies. Its general purpose scripting language is based on the simple yet powerful Lua programming language (with a few extensions) and embeds the state-of-art Just-In-Time compiler LuaJIT. Its physics is based on symplectic integration of differential maps made out of GTPSA (Generalized Truncated Power Series). The physics of the transport maps and the normal form analysis were both strongly inspired by the PTC/FPP library from E. Forest. MAD-NG development started in 2016 by the author as a side project of MAD-X, hence MAD-X users should quickly become familiar with its ecosystem, e.g. lattices definition.

MAD-NG is free open-source software, distributed under the GNU General Public License v3.[1] The source code, units tests,[2] integration tests, and examples are all available on its Github repository, including the documentation and its LaTeX source. For convenience, the binaries and few examples are also made available from the releases repository located on the AFS shared file system at CERN.

2 Installation

Download the binary corresponding to your platform from the releases repository and install it in a local directory. Update (or check) that the `PATH` environment variable contains the path to your local directory or prefix `mad` with this path to run it. Rename the application from `mad-`*`arch`*`-`*`v.m.n`* to `mad` and make it executable with the command '`chmod u+x mad`' on Unix systems or add the `.exe` extension on Windows.

```
$ ./mad -h
usage:  ./mad [options]...  [script [args]...].
Available options are:
  -e chunk   Execute string 'chunk'.
  -l name    Require library 'name'.
  -b ...     Save or list bytecode.
  -j cmd     Perform JIT control command.
  -O[opt]    Control JIT optimizations.
  -i         Enter interactive mode after executing 'script'.
  -q         Do not show version information.
  -M         Do not load MAD environment.
  -Mt[=num]  Set initial MAD trace level to 'num'.
  -MT[=num]  Set initial MAD trace level to 'num' and location.
  -E         Ignore environment variables.
  --         Stop handling options.
  -          Execute stdin and stop handling options.
```

[1] MAD-NG embeds the libraries FFTW, NFFT and NLopt released under GNU (L)GPL too.
[2] MAD-NG has few thousands unit tests that do few millions checks, and it is constantly growing.

2.1 Releases version

MAD-NG releases are tagged on the Github repository and use mangled binary names on the releases repository, i.e. `mad-`*`arch`*`-`*`v.m.n`* where:

`arch` is the platform architecture for binaries among `linux`, `macos` and `windows`.
`v` is the **v**ersion number, `0` meaning beta-version under active development.
`m` is the **m**ajor release number corresponding to features completeness.
`n` is the mi**n**or release number corresponding to bug fixes.

3 Interactive Mode

To run MAD-NG in interactive mode, just typewrite its name on the Shell invite like any command-line tool. It is recommended to wrap MAD-NG with the readline wrapper `rlwrap` in interactive mode for easier use and commands history:

```
$ rlwrap ./mad
   ____  __   ______    ______     |   Methodical Accelerator Design
    /  \/  \   /  _  \   /  _  \   |   release: 0.9.0 (OSX 64)
   /  __   /  /  /_/ /  /  /_/ /   |   support: http://cern.ch/mad
  /__/  /_/  /__/ /_/  /_____ /    |   licence: GPL3 (C) CERN 2016+
                                   |   started: 2020-08-01 20:13:51

 > print "hello world!"
 "hello world!"
```

Here the application is assumed to be installed in the current directory '`.`' and the character '>' is the prompt waiting for user input in interactive mode. If you write an incomplete statement, the interpreter waits for its completion by issuing a different prompt:

```
> print                -- 1st level prompt, incomplete statement
>> "hello world!"      -- 2nd level prompt, complete the statement
hello world!           -- execute
```

Typing the character '=' right after the 1st level prompt is equivalent to call the **`print`** function:

```
> = "hello world!"     -- 1st level prompt followed by =
hello world!           -- execute print "hello world!"
> = MAD.option.numfmt
% -.10g
```

To quit the application typewrite `Crtl+D` to send `EOF` (end-of-file) on the input,[3] or `Crtl+\` to send the `SIGQUIT` (quit) signal, or `Crtl+C` to send the stronger `SIGINT` (interrupt) signal. If the application is stalled or looping for ever, typewriting a single `Crtl+\` or `Crtl+C` twice will stop it:

```
> while true do end    -- loop forever, 1st Crtl+C doesn't stop it
pending interruption in VM! (next will exit)         -- 2nd Crtl+C
interrupted!           -- application stopped

> while true do end    -- loop forever, a single Crtl+\ does stop it
Quit: 3                -- Signal 3 caught, application stopped
```

[3]Note that sending `Crtl+D` twice from MAD-NG invite will quit both MAD-NG and its parent Shell. . .

In interactive mode, each line input is run in its own *chunk*[4], which also rules variables scopes. Hence **local** variables are not visible between chuncks, i.e. input lines. The simple solutions are either to use global variables or to enclose local statements into the same chunk delimited by the **do** ... **end** keywords:

```
> local a = "hello"
> print(a.." world!")
  stdin:1: attempt to concatenate global 'a' (a nil value)
  stack traceback:
  stdin:1: in main chunk
  [C]: at 0x01000325c0

> do                      -- 1st level prompt, open the chunck
>> local a = "hello"      -- 2nd level prompt, waiting for statement completion
>> print(a.." world!")    -- same chunk, local 'a' is visible
>> end                    -- close and execute the chunk
hello world!
> print(a)                -- here 'a' is an unset global variable
nil
> a = "hello"             -- set global 'a'
> print(a.." world!")     -- works but pollutes the global environment
hello world!
```

4 Batch Mode

To run MAD-NG in batch mode, just run it in the shell with files as arguments on the command line:

```
$ ./mad [mad options] myscript1.mad myscript2.mad ...
```

where the scripts contains programs written in the MAD-NG programming language (see Scripting).

5 Online Help

MAD-NG is equipped with an online help system[5] useful in interactive mode to quickly search for information displayed in the man-like Unix format :

```
> help()
Related topics:
MADX, aperture, beam, cmatrix, cofind, command, complex, constant, correct,
ctpsa, cvector, dynmap, element, filesys, geomap, gfunc, gmath, gphys, gplot,
gutil, hook, lfun, linspace, logrange, logspace, match, matrix, mflow,
monomial, mtable, nlogrange, nrange, object, operator, plot, range, reflect,
regex, sequence, strict, survey, symint, symintc, tostring, totable, tpsa,
track, twiss, typeid, utest, utility, vector.

> help "MADX"
NAME
MADX environment to emulate MAD-X workspace.
```

[4] A chunk is the unit of execution in Lua (see Lua 5.2 §3.3.2).

[5] The online help is far incomplete and will be completed, updated and revised as the application evolves.

```
    SYNOPSIS
    local lhcb1 in MADX

    DESCRIPTION
    This module provide the function 'load' that read MADX sequence and optics
    files and load them in the MADX global variable. If it does not exist, it will
    create the global MADX variable as an object and load into it all elements,
    constants, and math functions compatible with MADX.

    RETURN VALUES
    The MADX global variable.

    EXAMPLES
    MADX:open()
    -- inline definition
    MADX:close()

    SEE ALSO
    element, object.
```

Complementary to the `help` function, the function `show` displays the type and value of variables, and if they have attributes, the list of their names in the lexicographic order:

```
> show "hello world!"
:string: hello world!
> show(MAD.option)
:table: MAD.option
colwidth           :number: 18
hdrwidth           :number: 18
intfmt             :string: % -10d
madxenv            :boolean: false
nocharge           :boolean: false
numfmt             :string: % -.10g
ptcmodel           :boolean: false
strfmt             :string: % -25s
```

Chapter 2. Scripting

The choice of the scripting language for MAD-NG was sixfold: the *simplicity* and the *completeness* of the programming language, the *portability* and the *efficiency* of the implementation, and its easiness to be *extended* and *embedded* in an application. In practice, very few programming languages and implementations fulfill these requirements, and Lua and his Just-In-Time (JIT) compiler LuaJIT were not only the best solutions but almost the only ones available when the development of MAD-NG started in 2016.

1 Lua and LuaJIT

The easiest way to shortly describe these choices is to cite their authors.

"Lua is a powerful, efficient, lightweight, embeddable scripting language. It supports procedural programming, object-oriented programming, functional programming, data-driven programming, and data description. Lua combines simple procedural syntax with powerful data description constructs based on associative arrays and extensible semantics. Lua is dynamically typed and has automatic memory management with incremental garbage collection, making it ideal for configuration, scripting, and rapid prototyping."[1]

"LuaJIT is widely considered to be one of the fastest dynamic language implementations. It has outperformed other dynamic languages on many cross-language benchmarks since its first release in 2005 — often by a substantial margin — and breaks into the performance range traditionally reserved for offline, static language compilers."[2]

Lua and LuaJIT are free open-source software, distributed under the very liberal MIT license.

MAD-NG embeds a patched version of LuaJIT 2.1, a very efficient implementation of Lua 5.2.[3] Hence, the scripting language of MAD-NG is Lua 5.2 with some extensions detailed in the next section, and used for both, the development of most parts of the application, and as the user scripting language. There is no strong frontier between these two aspects of the application, giving full access and high flexibility to the experienced users. The filename extension of MAD-NG scripts is `.mad`.

Learning Lua is easy and can be achieved within a few hours. The following links should help to quickly become familiar with Lua and LuaJIT:

- Lua website.
- Lua 5.2 manual for MAD-NG (30 p. PDF).
- Lua 5.0 free online book (old).
- LuaJIT website.
- LuaJIT wiki.
- LuaJIT 2.1 documentation.
- LuaJIT 2.1 on GitHub.

2 Lua primer

The next subsections introduce the basics of the Lua programming language with syntax highlights, namely variables, control flow, functions, tables and methods.[4]

[1]This text is taken from the "What is Lua?" section of the Lua website.
[2]This text is taken from the "Overview" section of the LuaJIT website.
[3]The `ENV` feature of Lua 5.2 is not supported and will never be according to M. Pall.
[4]This primer was adapted from the blog "Learn Lua in 15 minutes" by T. Neylon.

2.1 Variables

```
n = 42  -- All numbers are doubles, but the JIT may specialize them.
-- IEEE-754 64-bit doubles have 52 bits for storing exact int values;
-- machine precision is not a problem for ints < 1e16.

s = 'walternate'  -- Immutable strings like Python.
t = "double-quotes are also fine"
u = [[ Double brackets
       start and end
       multi-line strings.]]
v = "double-quotes \z

     are also fine" -- \z eats next whitespaces
t, u, v = nil  -- Undefines t, u, v.
-- Lua has multiple assignments and nil completion.
-- Lua has garbage collection.

-- Undefined variables return nil. This is not an error:
foo = anUnknownVariable  -- Now foo = nil.
```

2.2 Control flow

```
-- Blocks are denoted with keywords like do/end:
while n < 50 do
  n = n + 1  -- No ++ or += type operators.
end

-- If clauses:
if n > 40 then
  print('over 40')
elseif s ~= 'walternate' then  -- ~= is not equals.
  -- Equality check is == like Python; ok for strs.
  io.write('not over 40\n')  -- Defaults to stdout.
else
  -- Variables are global by default.
  thisIsGlobal = 5  -- Camel case is common.
  -- How to make a variable local:
  local line = io.read()  -- Reads next stdin line.
  -- String concatenation uses the .. operator:
  print('Winter is coming, '..line)
end

-- Only nil and false are falsy; 0 and '' are true!
aBoolValue = false
if not aBoolValue then print('was false') end

-- 'or' and 'and' are short-circuited.
-- This is similar to the a?b:c operator in C/js:
ans = aBoolValue and 'yes' or 'no'  --> ans = 'no'

-- numerical for begin, end[, step] (end included)
```

```
revSum = 0
for j = 100, 1, -1 do revSum = revSum + j end
```

2.3 Functions

```
function fib(n)
  if n < 2 then return 1 end
  return fib(n - 2) + fib(n - 1)
end

-- Closures and anonymous functions are ok:
function adder(x)
  -- The returned function is created when adder is
  -- called, and captures the value of x:
  return function (y) return x + y end
end
a1 = adder(9)
a2 = adder(36)
print(a1(16))  --> 25
print(a2(64))  --> 100

-- Returns, func calls, and assignments all work with lists
-- that may be mismatched in length.
-- Unmatched receivers get nil; unmatched senders are discarded.

x, y, z = 1, 2, 3, 4
-- Now x = 1, y = 2, z = 3, and 4 is thrown away.

function bar(a, b, c)
  print(a, b, c)
  return 4, 8, 15, 16, 23, 42
end

x, y = bar('zaphod')  --> prints "zaphod  nil nil"
-- Now x = 4, y = 8, values 15,..,42 are discarded.

-- Functions are first-class, may be local/global.
-- These are the same:
function f(x) return x * x end
f = function (x) return x * x end

-- And so are these:
local function g(x) return math.sin(x) end
local g; g  = function (x) return math.sin(x) end
-- the 'local g' decl makes g-self-references ok.

-- Calls with one string param don't need parens:
print 'hello'  -- Works fine.
```

2.4 Tables

```
-- Tables = Lua's only compound data structure;
--   they are associative arrays, i.e. hash-lookup dicts;
--   they can be used as lists, i.e. sequence of non-nil values.

-- Dict literals have string keys by default:
t = {key1 = 'value1', key2 = false, ['key.3'] = true }

-- String keys looking as identifier can use dot notation:
print(t.key1, t['key.3']) -- Prints 'value1 true'.
-- print(t.key.3)         -- Error, needs explicit indexing by string
t.newKey = {}             -- Adds a new key/value pair.
t.key2 = nil              -- Removes key2 from the table.

-- Literal notation for any (non-nil) value as key:
u = {['@!#'] = 'qbert', [{}] = 1729, [6.28] = 'tau'}
print(u[6.28])  -- prints "tau"

-- Key matching is basically by value for numbers
-- and strings, but by identity for tables.
a = u['@!#']  -- Now a = 'qbert'.
b = u[{}]     -- We might expect 1729, but it's nil:

-- A one-table-param function call needs no parens:
function h(x) print(x.key1) end
h{key1 = 'Sonmi~451'}  -- Prints 'Sonmi~451'.

for key, val in pairs(u) do -- Table iteration.
  print(key, val)
end

-- List literals implicitly set up int keys:
l = {'value1', 'value2', 1.21, 'gigawatts'}
for i,v in ipairs(l) do  -- List iteration.
  print(i,v,l[i])        -- Indices start at 1 !
end
print("length=", #l)     -- # is defined only for sequence.
-- A 'list' is not a real type, l is just a table
-- with consecutive integer keys, treated as a list,
-- i.e. l = {[1]='value1', [2]='value2', [3]=1.21, [4]='gigawatts'}
-- A 'sequence' is a list with non-nil values.
```

2.5 Methods

```
-- Methods notation:
--   function tblname:fn(...) is the same as
--     function tblname.fn(self, ...) with self being the table.
--   calling tblname:fn(...) is the same as
--     tblname.fn(tblname, ...)       here self becomes the table.
t = { disp=function(s) print(s.msg) end, -- Method 'disp'
      msg="Hello world!" }
t:disp() -- Prints "Hello world!"
function t:setmsg(msg) self.msg=msg end  -- Add a new method 'setmsg'
```

```
t:setmsg "Good bye!"
t:disp() -- Prints "Good bye!"
```

3 Extensions

The aim of the extensions patches applied to the embedded LuaJIT in MAD-NG is to extend the Lua syntax in handy directions, like for example to support the deferred expression operator. A serious effort has been put to develop a Domain Specific Language (DSL) embedded in Lua using these extensions and the native language features to mimic as much as possible the syntax of MAD-X in the relevant aspects of the language, like the definition of elements, lattices or commands, and ease the transition of MAD-X users.

Bending and extending a programming language like Lua to embed a DSL is more general and challenging than creating a freestanding DSL like in MAD-X. The former is compatible with the huge codebase written by the Lua community, while the latter is a highly specialized niche language. The chosen approach attempts to get the best of the two worlds.

3.1 Line comment

The line comment operator `!` is valid in MAD-NG, but does not exists in Lua:[5]

```
local a = 1     ! this remaining part is a comment
local b = 2     -- line comment in Lua
```

3.2 Unary plus

The unary plus operator + is valid in MAD-NG, but does not exists in Lua:[5]

```
local a = +1    -- syntax error in Lua
local b = +a    -- syntax error in Lua
```

3.3 Local in table

The local **`in`** table syntax provides a convenient way to retrieve values from a *mappable* and avoid error-prone repetitions of attributes names. The syntax is as follows:

```
local sin, cos, tan in math      -- syntax error in Lua
local a, b, c in { a=1, b=2, c=3 }
! a, b, c in { a=1, b=2, c=3 }   -- invalid with global variables
```

which is strictly equivalent to the Lua code:

```
local sin, cos, tan = math.sin, math.cos, math.tan
local tbl = { a=1, b=2, c=3 }
local a, b, c = tbl.a, tbl.b, tbl.c
! local sin, cos, tan = math.cos, math.sin, math.tan   -- nasty typo
```

The JIT has many kinds of optimization to improve a lot the execution speed of the code, and these work much better if variables are declared **`local`** with minimal lifespan. *This language extension is of first importance for writing fast clean code!*

[5]This feature was introduced to ease the automatic translation of lattices from MAD-X to MAD-NG.

3.4 Lambda function

The lambda function syntax is pure syntactic sugar for function definition and therefore fully compatible with the Lua semantic. The following definitions are all semantically equivalent:

```
local f = function(x) return x^2 end  -- Lua syntax
local f = \x x^2                      -- most compact form
local f = \x -> x^2                   -- most common form
local f = \(x) -> x^2                 -- for readability
local f = \(x) -> (x^2)               -- less compact form
local f = \x (x^2)                    -- uncommon valid form
local f = \(x) x^2                    -- uncommon valid form
local f = \(x) (x^2)                  -- uncommon valid form
```

The important point is that no space must be present between the *lambda* operator \ and the first formal parameter or the first parenthesis; the former will be considered as an empty list of parameters and the latter as an expressions list returning multiple values, and both will trigger a syntax error. For the sake of readability, it is possible without changing the semantic to add extra spaces anywhere in the definition, add an arrow operator ->, or add parentheses around the formal parameter list, whether the list is empty or not.

The following examples show *lambda* functions with multiple formal parameters:

```
local f = function(x,y) return x+y end  -- Lua syntax
local f = \x x+y                        -- most compact form
local f = \x,y -> x+y                   -- most common form
local f = \x, y -> x + y                -- aerial style
```

The lambda function syntax supports multiple return values by enclosing the list of returned expressions within (not optional!) parentheses:

```
local f = function(x,y) return x+y, x-y end  -- Lua syntax
local f = \x,y(x+y,x-y)                      -- most compact form
local f = \x,y -> (x+y,x-y)                  -- most common form
```

Extra surrounding parentheses can also be added to disambiguate false multiple return values syntax:

```
local f = function(x,y) return (x+y)/2 end  -- Lua syntax
local f = \x,y -> ((x+y)/2)     -- disambiguation: single value returned
! local f = \x,y -> (x+y)/2     -- invalid syntax at '/'

local f = function(x,y) return (x+y)*(x-y) end -- Lua syntax
local f = \x,y -> ((x+y)*(x-y)) -- disambiguation: single value returned
! local f = \x,y -> (x+y)*(x-y) -- invalid syntax at '*'
```

It is worth understanding the error message that invalid syntaxes above would report,

```
file:line:  attempt to perform arithmetic on a function value.
```

as it is a bit subtle and needs some explanations: the *lambda* is syntactically closed at the end of the returned expression `(x+y)`, and the following operations `/` or `*` are considered as being outside the *lambda* definition, that is applied to the freshly created function itself...

Finally, the *lambda* function syntax supports full function syntax (for consistency) using the *fat* arrow operator => in place of the arrow operator:

```
local c = 0
local f = function(x) c=c+1 return x^2 end   -- Lua syntax
local f = \x => c=c+1 return x^2 end         -- most compact form
```

The fat arrow operator requires the **end** keyword to close syntactically the *lambda* function, and the **return** keyword to return values (if any), as in Lua functions definitions.

3.5 Deferred expression

The deferred expression operator `:=` is semantically equivalent to a *lambda* function without argument. It is syntactically valid only inside *table* constructors (see Lua 5.2 §3.4.8):[6]

```
local var = 10
local fun = \-> var
! local fun := var  -- invalid syntax outside table constructors
local tbl = { v1 := var, v2 =\-> var, v3 = var }
print(tbl.v1(), tbl.v2(), tbl.v3, fun()) -- display: 10 10 10 10
var = 20
print(tbl.v1(), tbl.v2(), tbl.v3, fun()) -- display: 20 20 10 20
```

The deferred expressions hereabove have to be explicitly called to retrieve their values, because they are defined in a *table*. It is a feature of the object model making the deferred expressions behaving like values. Still, it is possible to support deferred expressions as values in a raw *table*, i.e. a table without metatable, using the `deferred` function from the `typeid` module:

```
local deferred in MAD.typeid
local var = 10
local tbl = deferred { v1 := var, v2 =\-> var, v3 = var }
print(tbl.v1, tbl.v2, tbl.v3) -- display: 10 10 10
var = 20
print(tbl.v1, tbl.v2, tbl.v3) -- display: 20 20 10
```

3.6 Ranges

The ranges are created from pairs or triplets of concatenated numbers:[7]

```
start..stop..step   -- order is the same as numerical 'for'
start..stop         -- default step is 1
3..4                -- spaces are not needed around concat operator
3..4..0.1           -- floating numbers are handled
4..3..-0.1          -- negative steps are handled
stop..start..-step  -- operator precedence
```

The default value for unspecified `step` is `1`. The Lua syntax has been modified to accept concatenation operator without surrounding spaces for convenience.

Ranges are *iterable* and *lengthable* so the following code excerpt is valid:

```
local rng = 3..4..0.1
print(#rng) -- display: 11
for i,v in ipairs(rng) do print(i,v) end
```

More details on ranges can be found in the `Range` module, especially about the `range` and `logrange` constructors that may adjust `step` to ensure precise loops and iterators behaviors with floating-point numbers.

[6]This feature was introduced to ease the automatic translation of lattices from MAD-X to MAD-NG.

[7]This is the only feature of MAD-NG that is incompatible with the semantic of Lua.

3.7 Lua syntax and extensions

The operator precedence (see Lua 5.2 §3.4.7) is recapped and extended in Table 2.1 with their precedence level (on the left) from lower to higher priority and their associativity (on the right).

Table 2.1: Operators precedence with priority and associativity.

1:	`or`	left
2:	`and`	left
3:	`< > <= >= ~= ==`	left
4:	`..`	right
5:	`+ - (binary)`	left
6:	`* / %`	left
7:	`not # - + (unary)`	left
8:	`^`	right
9:	`. [] () (call)`	left

The *string* literals, *table* constructors, and *lambda* definitions can be combined with function calls (see Lua 5.2 §3.4.9) advantageously like in the object model to create objects in a similar way to MAD-X. The following function calls are semantically equivalent by pairs:

```
! with parentheses                    ! without parentheses
func( 'hello world!' )                func  'hello world!'
func( "hello world!" )                func  "hello world!"
func( [[hello world!]] )              func  [[hello world!]]
func( {...fields...} )                func  {...fields...}
func( \x -> x^2 )                     func  \x -> x^2
func( \x,y -> (x+y,x-y) )             func  \x,y -> (x+y,x-y)
```

4 Types

MAD-NG is based on Lua, a dynamically typed programming language that provides the following *basic types* often italicized in this textbook:

nil The type of the value `nil`. Uninitialized variables, unset attributes, mismatched arguments, mismatched return values etc, have `nil` values.

boolean The type of the values `true` and `false`.

number The type of IEEE 754 double precision floating point numbers. They are exact for integers up to $\pm 2^{53}$ ($\approx \pm 10^{16}$). Value like `0`, `1`, `1e3`, `1e-3` are numbers.

string The type of character strings. Strings are "internalized" meaning that two strings with the same content compare equal and share the same memory address:
`a="hello"; b="hello"; print(a==b) --display: true`.

table The type of tables, see Lua 5.2 §3.4.8 for details. In this textbook, the following qualified types are used to distinguish between two kinds of special use of tables:

- A *list* is a table used as an array, that is a table indexed by a *continuous* sequence of integers starting from `1` where the length operator `#` has defined behavior.[8]
- A *set* is a table used as a dictionnary, that is a table indexed by keys — strings or other types — or a *sparse* sequence of integers where the length operator `#` has undefined behavior.

function	The type of functions, see Lua 5.2 §3.4.10 for details. In this textbook, the following qualified types are used to distinguish between few kinds of special use of functions:

- A *lambda* is a function defined with the `\` syntax.
- A *functor* is an object[9] that behaves like a function.
- A *method* is a function called with the `:` syntax and its owner as first argument. A *method* defined with the `:` syntax has an implicit first argument named `self`.[10]

thread	The type of coroutines, see Lua 5.2 §2.6 for details.
userdata	The type of raw pointers with memory managed by Lua, and its companion *lightuserdata* with memory managed by the host language, usually C. They are mainly useful for interfacing Lua with its C API, but MAD-NG favors the faster FFI[11] extension of LuaJIT.
cdata	The type of C data structures that can be defined, created and manipulated directly from Lua as part of the FFI[11] extension of LuaJIT. The numeric ranges, the complex numbers, the (complex) matrices, and the (complex) GTPSA are *cdata* fully compatible with the embedded C code that operates them.

This textbook uses also some extra terms in place of types:

value	An instance of any type.
reference	A valid memory location storing some *value*.
logical	A *value* used by control flow, where `nil` $\equiv$ `false` and *anything-else* $\equiv$ `true`.

4.1 Value vs reference

The types *nil*, *boolean* and *number* have a semantic by *value*, meaning that variables, arguments, return values, etc., hold their instances directly. As a consequence, any assignment makes a copy of the *value*, i.e. changing the original value does not change the copy.

The types *string*, *function*, *table*, *thread*, *userdata* and *cdata* have a semantic by *reference*, meaning that variables, arguments, return values, etc., do not store their instances directly but a *reference* to them. As a consequence, any assignment makes a copy of the *reference* and the instance becomes shared, i.e. references have a semantic by *value* but changing the content of the value does change the copy.[12]

The types *string*, *function*[13], *thread*, `complex` *cdata* and numeric `(log)range` *cdata* have a hybrid semantic. In practice these types have a semantic by *reference*, but they behave like types with semantic by *value* because their instances are immutable, and therefore sharing them is safe.

[8]The Lua community uses the term *sequence* instead of *list*, which is confusing is the context of MAD-NG.
[9]Here the term "object" is used in the Lua sense, not as an object from the object model of MAD-NG.
[10]This *hidden* methods argument is named `self` in Lua and Python, or `this` in Java and C++.
[11]FFI stands for Foreign Function Interface, an acronym well known in high-level languages communities.
[12]References semantic in Lua is similar to pointers semantic in C, see ISO/IEC 9899:1999 §6.2.5.
[13]Local variables and upvalues of functions can be modified using the `debug` module.

The instances of the (basic) types with semantic by *reference* are called “objects” in Lua, and should not be confused with the instances of the *object* (non-basic) type introduced by the object model of MAD-NG.

5 Concepts

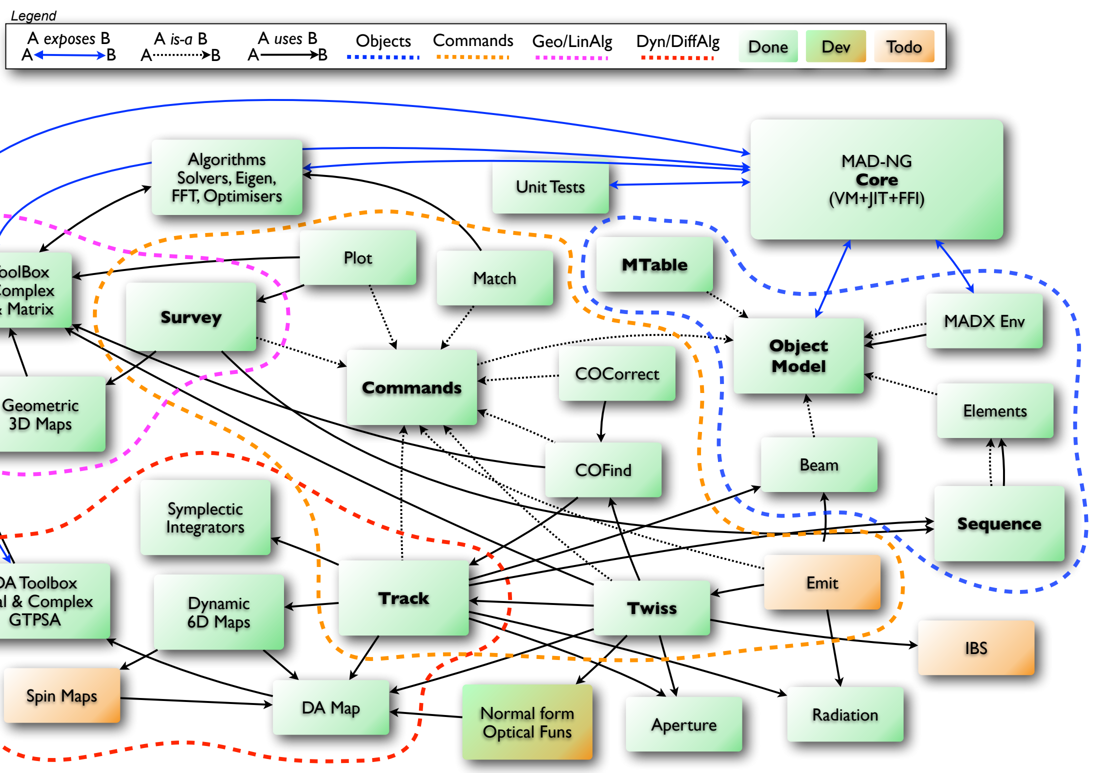
The concepts are natural extensions of types that concentrate more on behavior of objects[9] than on types. MAD-NG introduces many concepts to validate objects passed as argument before using them. The main concepts used in this textbook are listed below, see the `typeid` module for more concepts:

lengthable	An object that can be sized using the length operator `#`. Strings, lists, vectors and ranges are examples of *lengthable* objects.
indexable	An object that can be indexed using the square bracket operator `[]`. Tables, vectors and ranges are examples of *indexable* objects.
iterable	An object that can be iterated with a loop over indexes or a generic `for` with the `ipairs` iterator. Lists, vectors and ranges are examples of *iterable* objects.
mappable	An object that can be iterated with a loop over keys or a generic `for` with the `pairs` iterator. Sets and objects (from the object model) are examples of *mappable* objects.
callable	An object that can be called using the call operator `()`. Functions and functors are examples of *callable* objects.

6 Ecosystem

The Figure 2.1 shows a schematic representation of the ecosystem of MAD-NG, which should help the users to understand the relatioship between the different components of the application. The dashed lines are grouping the items (e.g. modules) by topics while the arrows are showing interdependencies between them and the colors their status.

Figure 2.1: MAD-NG ecosystem and status.



Chapter 3. Objects

The object model is of key importance as it implements many features used extensively by objects like `beam`, `sequence`, `mtable`, all the commands, all the elements, and the `MADX` environment. The aim of the object model is to extend the scripting language with concepts like objects, inheritance, methods, metamethods, deferred expressions, commands and more.

In computer science, the object model of MAD-NG is said to implement the concepts of prototypical objects, single inheritance and dynamic lookup of attributes:

- A *prototypical object* is an object created from a prototype,[1] named its parent.
- *Single inheritance* specifies that an object has only one direct parent.
- *Dynamic lookup* means that undefined attributes are searched in the parents at *each* read.

A prototype represents the default state and behavior, and new objects can reuse part of the knowledge stored in the prototype by inheritance, or by defining how the new object differs from the prototype. Because any object can be used as a prototype, this approach holds some advantages for representing default knowledge, and incrementally and dynamically modifying them.

1 Creation

The creation of a new object requires to hold a reference to its parent, i.e. the prototype, which indeed will create the child and return it as if it were returned from a function:

```
local object in MAD
local obj = object { }
```

The special *root object* `object` from the `MAD` environment is the parent of *all* objects, including elements, sequences, TFS tables and commands. It provides by inheritance the methods needed to handle objects, environments, and more. In this minimalist example, the created object has `object` as parent, so it is the simplest object that can be created.

It is possible to name immutably an object during its creation:

```
local obj = object 'myobj' { }
print(obj.name) -- display: myobj
```

Here,[2] `obj` is the variable holding the object while the *string* `'myobj'` is the name of the object. It is important to distinguish well the variable that holds the *object* from the object's name that holds the *string*, because they are very often named the same.

It is possible to define attributes during object creation or afterward:

```
local obj = object 'myobj' { a=1, b='hello' }
obj.c = { d=5 } -- add a new attribute c
print(obj.name, obj.a, obj.b, obj.c.d)  -- display: myobj 1 hello 5
```

1.1 Constructors

The previous object creation can be done equivalently using the prototype as a constructor:

```
local obj = object('myobj',{ a=1, b='hello' })
```

[1]Objects are not clones of prototypes, they share states and behaviors with their parents but do not hold copies.

[2]This syntax for creating objects eases the lattices translation from MAD-X to MAD-NG.

An object constructor expects two arguments, an optional *string* for the name, and a required *table* for the attributes placeholder, optionally filled with initial attributes. The table is used to create the object itself, so it cannot be reused to create a different object:

```
local attr = { a=1, b='hello' }
local obj1 = object('obj1',attr) -- ok
local obj2 = object('obj2',attr) -- runtime error, attr is already used.
```

The following objects creations are all semantically equivalent but use different syntax that may help to understand the creation process and avoid runtime errors:

```
-- named objects:
local nobj = object 'myobj'  { }  -- two stages creation.
local nobj = object 'myobj' ({ }) -- idem.
local nobj = object('myobj') { }  -- idem.
local nobj = object('myobj')({ }) -- idem.
local nobj = object('myobj', { }) -- one stage creation.
-- unnamed objects:
local uobj = object   { }         -- one stage creation.
local uobj = object  ({ })        -- idem.
local uobj = object() { }         -- two stages creation.
local uobj = object()({ })        -- idem.
local uobj = object(nil,{ })      -- one stage creation.
```

1.2 Incomplete objects

The following object creation shows how the two stage form can create an incomplete object that can only be used to complete its construction:

```
local obj = object 'myobj'   -- obj is incomplete, table is missing
print(obj.name)              -- runtime error.
obj = obj { }                -- now obj is complete.
print(obj.name)              -- display: myobj
```

Any attempt to use an incomplete object will trigger a runtime error with a message like:

```
file:line: forbidden read access to incomplete object.
```

or

```
file:line: forbidden write access to incomplete object.
```

depending on the kind of access.

1.3 Classes

An object used as a prototype to create new objects becomes a *class*, and a class cannot change, add, remove or override its methods and metamethods. This restriction ensures the behavioral consistency between the children after their creation. An object qualified as *final* cannot create instances and therefore cannot become a class.

1.4 Identification

The `object` module extends the `typeid` module with the `is_object(a)` *function*, which returns `true` if its argument `a` is an object, `false` otherwise:

```
local is_object in MAD.typeid
print(is_object(object), is_object(object{}), is_object{})
-- display: true  true  false
```

It is possible to know the objects qualifiers using the appropriate methods:

```
print(object:is_class(), object:is_final(), object:is_readonly())
-- display: true  false  true
```

1.5 Customizing creation

During the creation process of objects, the metamethod `__init(self)` is invoked if it exists, with the newly created object as its sole argument to let the parent finalize or customize its initialization *before* it is returned. This mechanism is used by commands to run their `:exec()` *method* during their creation.

2 Inheritance

The object model allows to build tree-like inheritance hierarchy by creating objects from classes, themselves created from other classes, and so on until the desired hierarchy is modeled. The example below shows an excerpt of the taxonomy of the elements as implemented by the `element` module, with their corresponding depth levels in comment:

```
local object in MAD                     -- depth level 1
local element = object            {...} -- depth level 2

local drift_element = element     {...} -- depth level 3
local instrument = drift_element {...} -- depth level 4
local monitor  = instrument       {...} -- depth level 5
local hmonitor = monitor          {...} -- depth level 6
local vmonitor = monitor          {...} -- depth level 6

local thick_element = element     {...} -- depth level 3
local tkicker = thick_element     {...} -- depth level 4
local kicker  = tkicker           {...} -- depth level 5
local hkicker = kicker            {...} -- depth level 6
local vicker  = kicker            {...} -- depth level 6
```

2.1 Reading attributes

Reading an attribute not defined in an object triggers a recursive dynamic lookup along the chain of its parents until it is found or the root `object` is reached. Reading an object attribute defined as a *function* automatically evaluates it with the object passed as the sole argument and the returned value is forwarded to the reader as if it were the attribute's value. When the argument is not used by the function, it becomes a *deferred expression* that can be defined directly with the operator `:=` as explained in section 3.5. This feature allows to use attributes holding values and functions the same way and postpone design decisions, e.g. switching from simple value to complex calculations without impacting the users side with calling parentheses at every use.

The following example is similar to the second example of the section 3.5, and it must be clear that `fun` must be explicitly called to retrieve the value despite that its definition is the same as the attribute `v2`.

```
local var = 10
local fun = \-> var -- here := is invalid
local obj = object { v1 := var, v2 =\-> var, v3 = var }
print(obj.v1, obj.v2, obj.v3, fun()) -- display: 10 10 10 10
var = 20
print(obj.v1, obj.v2, obj.v3, fun()) -- display: 20 20 10 20
```

2.2 Writing attributes

Writing to an object uses direct access and does not involve any lookup. Hence setting an attribute with a non-`nil` value in an object hides his definition inherited from the parents, while setting an attribute with `nil` in an object restores the inheritance lookup:

```
local obj1 = object { a=1, b='hello' }
local obj2 = obj1 { a=\s-> s.b..' world' }
print(obj1.a, obj2.a) -- display: 1 hello world
obj2.a = nil
print(obj1.a, obj2.a) -- display: 1 1
```

This property is extensively used by commands to specify their attributes default values or to rely on other commands attributes default values, both being overridable by the users.

It is forbidden to write to a read-only objects or to a read-only attributes. The former can be set using the `:readonly` *method*, while the latter corresponds to attributes with names that start by `__`, i.e. two underscores.

2.3 Class instances

To determine if an object is an instance of a given class, use the `:is_instanceOf` *method*:

```
local hmonitor, instrument, element in MAD.element
print(hmonitor:is_instanceOf(instrument)) -- display: true
```

To get the list of *public* attributes of an instance, use the `:get_varkeys`*method*:

```
for _,a in ipairs(hmonitor:get_varkeys()) do print(a) end
for _,a in ipairs(hmonitor:get_varkeys(object)) do print(a) end
for _,a in ipairs(hmonitor:get_varkeys(instrument)) do print(a) end
for _,a in ipairs(element:get_varkeys()) do print(a) end
```

The code snippet above lists the names of the attributes set by:

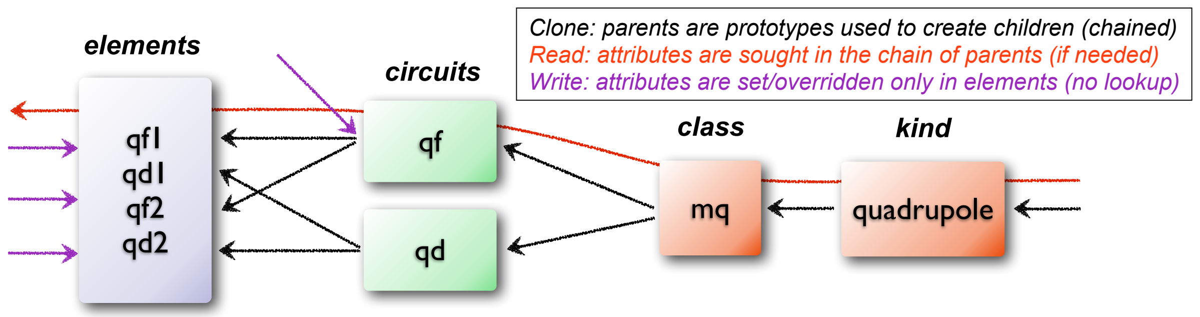
- the object `hmonitor` (only).
- the objects in the hierachy from `hmonitor` to `object` included.
- the objects in the hierachy from `hmonitor` to `instrument` included.
- the object `element` (only), the root of all elements.

2.4 Examples

The Figure 3.1 summarizes inheritance and attributes lookup with arrows and colors, which are reproduced by the example hereafter:

```
local element, quadrupole in MAD.element    -- kind
local mq  = quadrupole 'mq'  { l  =  2.1  } -- class
local qf  = mq         'qf'  { k1 =  0.05 } -- circuit
local qd  = mq         'qd'  { k1 = -0.06 } -- circuit
```

Figure 3.1: Object model and inheritance.


Clone: parents are prototypes used to create children (chained)
Read: attributes are sought in the chain of parents (if needed)
Write: attributes are set/overridden only in elements (no lookup)
elements
qf1
qd1
qf2
qd2
circuits
qf
qd
class
mq
kind
quadrupole

```
local qf1 = qf          'qf1' {}             -- element
... -- more elements
print(qf1.k1)                      -- display: 0.05 (lookup)
qf.k1 = 0.06                       -- update strength of 'qf' circuit
print(qf1.k1)                      -- display: 0.06 (lookup)
qf1.k1 = 0.07                      -- set strength of 'qf1' element
print(qf.k1, qf1.k1)               -- display: 0.06 0.07 (no lookup)
qf1.k1 = nil                       -- cancel strength of 'qf1' element
print(qf1.k1, qf1.l)               -- display: 0.06 2.1 (lookup)
print(#element:get_varkeys())      -- display: 33 (may vary)
```

The element `quadrupole` provided by the `element` module is the father of the objects created on its left. The *black arrows* show the user defined hierarchy of object created from and linked to the `quadrupole`. The main quadrupole `mq` is a user class representing the physical element, e.g. defining a length, and used to create two new classes, a focusing quadrupole `qf` and a defocusing quadrupole `qd` to model the circuits, e.g. hold the strength of elements connected in series, and finally the real individual elements `qf1`, `qd1`, `qf2` and `qd2` that will populate the sequence. A tracking command will request various attributes when crossing an element, like its length or its strength, leading to lookup of different depths in the hierarchy along the *red arrow*. A user may also write or overwrite an attribute at different level in the hierarchy by accessing directly to an element, as shown by the *purple arrows*, and mask an attribute of the parent with the new definitions in the children. The construction shown in this example follows the *separation of concern* principle and it is still highly reconfigurable despite that is does not contain any deferred expression or lambda function.

3 Attributes

New attributes can be added to objects using the dot operator `.` or the indexing operator `[]` as for tables. Attributes with non-*string* keys are considered as private. Attributes with *string* keys starting by two underscores are considered as private and read-only, and must be set during creation:

```
mq.comment = "Main Arc Quadrupole"
print(qf1.comment)       -- displays: Main Arc Quadrupole
qf.__k1 = 0.01           -- error
qf2 = qf { __k1=0.01 }   -- ok
```

The root `object` provides the following attributes:

name A *lambda* returning the *string* `__id`.

parent A *lambda* returning a *reference* to the parent *object*.

Warning: the following private and read-only attributes are present in all objects as part of the object model and should *never be used, set or changed*; breaking this rule would lead to an *undefined behavior*:

`__id`	A *string* holding the object's name set during its creation.
`__par`	A *reference* holding the object's parent set during its creation.
`__flg`	A *number* holding the object's flags.
`__var`	A *table* holding the object's variables, i.e. pairs of (*key*, *value*).
`__env`	A *table* holding the object's environment.
`__index`	A *reference* to the object's parent variables.

4 Methods

New methods can be added to objects but not classes, using the `:set_methods(set)` *method* with `set` being the *set* of methods to add as in the following example:

```
sequence :set_methods {
  name_of   = name_of,
  index_of  = index_of,
  range_of  = range_of,
  length_of = length_of,
  ...
}
```

where the keys are the names of the added methods and their values must be a *callable* accepting the object itself, i.e. `self`, as their first argument. Classes cannot set new methods.

The root `object` provides the following methods:

`is_final`	A *method* `()` returning a *boolean* telling if the object is final, i.e. cannot have instance.
`is_class`	A *method* `()` returning a *boolean* telling if the object is a *class*, i.e. had/has an instance.
`is_readonly`	A *method* `()` returning a *boolean* telling if the object is read-only, i.e. attributes cannot be changed.
`is_instanceOf`	A *method* `(cls)` returning a *boolean* telling if `self` is an instance of `cls`.
`set_final`	A *method* `([a])` returning `self` set as final if `a ~= false` or non-final.
`set_readonly`	A *method* `([a])` returning `self` set as read-only if `a ~= false` or read-write.
`same`	A *method* `([name])` returning an empty clone of `self` and named after the *string* `name` (default: `nil`).
`copy`	A *method* `([name])` returning a copy of `self` and named after the *string* `name` (default: `nil`). The private attributes are not copied, e.g. the final, class or read-only qualifiers are not copied.
`get_varkeys`	A *method* `([cls])` returning both, the *list* of the non-private attributes of `self` down to `cls` (default: `self`) included, and the *set* of their keys in the form of pairs (*key*, *key*).

get_variables A *method* `(lst, [set], [noeval])` returning a *set* containing the pairs (*key*, *value*) of the attributes listed in `lst`. If `set` is provided, it will be used to store the pairs. If `noveval == true`, the functions are not evaluated. The full *list* of attributes can be retrieved from `get_varkeys`. Shortcut `getvar`.

set_variables A *method* `(set, [override])` returning `self` with the attributes set to the pairs (*key*, *value*) contained in `set`. If `override ~= true`, the read-only attributes (with *key* starting by `"__"`) cannot be updated.

copy_variables A *method* `(set, [lst], [override])` returning `self` with the attributes listed in `lst` set to the pairs (*key*, *value*) contained in `set`. If `lst` is not provided, it is replaced by `self.__attr`. If `set` is an *object* and `lst.noeval` exists, it is used as the list of attributes to copy without function evaluation.[3] If `override ~= true`, the read-only attributes (with *key* starting by `"__"`) cannot be updated. Shortcut `cpyvar`.

wrap_variables A *method* `(set, [override])` returning `self` with the attributes wrapped by the pairs (*key*, *value*) contained in `set`, where the *value* must be a *callable* `(a)` that takes the attribute (as a callable) and returns the wrapped *value*. If `override ~= true`, the read-only attributes (with *key* starting by `"__"`) cannot be updated.

The following example shows how to convert the length `l` of an RBEND from cord to arc,[4] keeping its strength `k0` to be computed on the fly:

```
local cord2arc in MAD.gmath
local rbend    in MAD.element
local printf   in MAD.utility
local rb = rbend 'rb' { angle=pi/10, l=2, k0=\s s.angle/s.l }
printf("l=%.5f, k0=%.5f\n", rb.l, rb.k0) -- l=2.00000, k0=0.15708
rb:wrap_variables { l=\l\s cord2arc(l(),s.angle) } -- RBARC
printf("l=%.5f, k0=%.5f\n", rb.l, rb.k0) -- l=2.00825, k0=0.15643
rb.angle = pi/20 -- update angle
printf("l=%.5f, k0=%.5f\n", rb.l, rb.k0) -- l=2.00206, k0=0.07846
```

The method converts non-*callable* attributes into callables automatically to simplify the user-side, i.e. `l()` can always be used as a *callable* whatever its original form was. At the end, `k0` and `l` are computed values and updating `angle` affects both as expected.

clear_variables A *method* `()` returning `self` after setting all non-private attributes to `nil`.

clear_array A *method* `()` returning `self` after setting the array slots to `nil`, i.e. clear the *list* part.

clear_all A *method* `()` returning `self` after clearing the object except its private attributes.

set_methods A *method* `(set, [override])` returning `self` with the methods set to the pairs (*key*, *value*) contained in `set`, where *key* must be a *string* (the method's name) and *value* must be a *callable* (the method itself). If `override ~= true`, the read-only methods (with *key* starting by `"__"`) cannot be updated. Classes cannot update their methods.

set_metamethods A *method* `(set, [override])` returning `self` with the attributes set to the pairs (*key*, *value*) contained in `set`, where *key* must be a *string* (the metamethod's name) and *value* must be a *callable* (the metamethod itself). If `override == false`, the metamethods cannot be updated. Classes cannot update their metamethods.

[3]This feature is used to setup a command from another command, e.g. `track` from `twiss`.

[4]This approach is safer than the volatile option `RBARC` of MAD-X.

`insert`	A *method* `([idx], a)` returning `self` after inserting `a` at the position `idx` (default: `#self+1`) and shifting up the items at positions `idx..`.
`remove`	A *method* `([idx])` returning the *value* removed at the position `idx` (default: `#self`) and shifting down the items at positions `idx..`.
`move`	A *method* `(idx1, idx2, idxto, [dst])` returning the destination object `dst` (default: `self`) after moving the items from `self` at positions `idx1..idx2` to `dst` at positions `idxto..`. The destination range can overlap with the source range.
`sort`	A *method* `([cmp])` returning `self` after sorting in-place its *list* part using the ordering *callable* `cmp(`a_i`, `a_j`)` (default: `"<"`), which must define a partial order over the items. The sorting algorithm is not stable.
`bsearch`	A *method* `(a, [cmp], [low], [high])` returning the lowest index `idx` in the range specified by `low..high` (default: `1..#self`) from the **ordered** *list* of `self` that compares `true` with item `a` using the *callable* `cmp(a, self[idx])` (default: `"<="` for ascending, `">="` for descending), or `high+1`. In the presence of multiple equal items, `"<="` (resp. `">="`) will return the index of the first equal item while `"<"` (resp. `">"`) the index next to the last equal item for ascending (resp. descending) order.[5]
`lsearch`	A *method* `(a, [cmp], [low], [high])` returning the lowest index `idx` in the range specified by `low..high` (default: `1..#self`) from the *list* of `self` that compares `true` with item `a` using the *callable* `cmp(a, self[idx])` (default: `"=="`), or `high+1`. In the presence of multiple equal items in an ordered *list*, `"<="` (resp. `">="`) will return the index of the first equal item while `"<"` (resp. `">"`) the index next to the last equal item for ascending (resp. descending) order.[5]
`get_flags`	A *method* `()` returning the flags of `self`. The flags are not inherited nor copied.
`set_flags`	A *method* `(flgs)` returning `self` after setting the flags determined by `flgs`.
`clear_flags`	A *method* `(flgs)` returning `self` after clearing the flags determined by `flgs`.
`test_flags`	A *method* `(flgs)` returning a *boolean* telling if all the flags determined by `flgs` are set.
`open_env`	A *method* `([ctx])` returning `self` after opening an environment, i.e. a global scope, using `self` as the context for `ctx` (default: 1). The argument `ctx` must be either a *function* or a *number* defining a call level $\geqslant 1$.
`close_env`	A *method* `()` returning `self` after closing the environment linked to it. Closing an environment twice is safe.
`load_env`	A *method* `(loader)` returning `self` after calling the `loader`, i.e. a compiled chunk, using `self` as its environment. If the loader is a *string*, it is interpreted as the filename of a script to load, see functions `load` and `loadfile` in Lua 5.2 §6.1 for details.
`dump_env`	A *method* `()` returning `self` after dumping its content on the terminal in the rought form of pairs (*key*, *value*), including content of table and object *value*, useful for debugging environments.
`is_open_env`	A *method* `()` returning a *boolean* telling if `self` is an open environment.

[5] `bsearch` and `lsearch` stand for binary (ordered) search and linear (unordered) search respectively.

raw_len A *method* `()` returning the *number* of items in the *list* part of the object. This method should not be confused with the native *function* `rawlen`.

raw_get A *method* `(key)` returning the *value* of the attribute `key` without *lambda* evaluation nor inheritance lookup. This method should not be confused with the native *function* `rawget`.

raw_set A *method* `(key, val)` setting the attribute `key` to the *value* `val`, bypassing all guards of the object model. This method should not be confused with the native *function* `rawset`. **Warning**: use this dangerous method at your own risk!

var_get A *method* `(key)` returning the *value* of the attribute `key` without *lambda* evaluation.

var_val A *method* `(key, val)` returning the *value* `val` of the attribute `key` with *lambda* evaluation. This method is the complementary of `var_get`, i.e. `__index` $\equiv$ `var_val` $\circ$ `var_get`.

dumpobj A *method* `([fname], [cls], [patt], [noeval])` return `self` after dumping its non-private attributes in file `fname` (default: `stdout`) in a hierarchical form down to `cls`. If the *string* `patt` is provided, it filters the names of the attributes to dump. If `fname == '-'`, the dump is returned as a *string* in place of `self`. The *logical* `noeval` prevents the evaluatation the deferred expressions and reports the functions addresses instead. In the output, `self` and its parents are displayed indented according to their inheritance level, and preceeded by a `+` sign. The attributes overridden through the inheritance are tagged with n $*$ signs, where n corresponds to the number of overrides since the first definition.

5 Metamethods

New metamethods can be added to objects but not classes, using the `:set_metamethods(set)` *method* with `set` being the *set* of metamethods to add as in the following example:

```
sequence :set_metamethods {
  __len      = len_mm,
  __index    = index_mm,
  __newindex = newindex_mm,
  ...
}
```

where the keys are the names of the added metamethods and their values must be *callable* accepting the object itself, i.e. `self`, as their first argument. Classes cannot set new metamethods.

The root `object` provides the following metamethods:

__init A *metamethod* `()` called to finalize `self` before returning from the constructor.

__same A *metamethod* `()` similar to the *method* `same`.

__copy A *metamethod* `()` similar to the *method* `copy`.

__len A *metamethod* `()` called by the length operator `#` to return the size of the *list* part of `self`.

__call A *metamethod* `([name], tbl)` called by the call operator `()` to return an instance of `self` created from `name` and `tbl`, i.e. using `self` as a constructor.

__index	A *metamethod* `(key)` called by the indexing operator `[key]` to return the *value* of an attribute determined by *key* after having performed *lambda* evaluation and inheritance lookup.
__newindex	A *metamethod* `(key, val)` called by the assignment operator `[key]=val` to create new attributes for the pairs (*key*, *value*).
__pairs	A *metamethod* `()` called by the `pairs` *function* to return an iterator over the non-private attributes of `self`.
__ipairs	A *metamethod* `()` called by the `ipairs` *function* to return an iterator over the *list* part of `self`.
__tostring	A *metamethod* `()` called by the `tostring` *function* to return a *string* describing succinctly `self`.

The following attributes are stored with metamethods in the metatable, but have different purposes:

__obj	A unique private *reference* that characterizes objects.
__metatable	A *reference* to the metatable itself protecting against modifications.

6 Flags

The object model uses *flags* to qualify objects, like *class*-object, *final*-object and *readonly*-object. The difference with *boolean* attributes is that flags are *not* inherited nor copied. The flags of objects are managed by the methods `:get_flags`, `:set_flags`, `:clear_flags` and `:test_flags`. Methods like `:is_class`, `:is_final` and `:is_readonly` are roughly equivalent to call the method `:test_flags` with the corresponding (private) flag as argument. Note that functions from the `typeid` module that check for types or kinds, like `is_object` or `is_beam`, never rely on flags because types and kinds are not qualifers.

From the technical point of view, flags are encoded into a 32-bit integer and the object model uses the protected bits 29-31, hence bits 0-28 are free of use. Object flags can be used and extended by other modules introducing their own flags, like the `element` module that relies on bits 0-4 and used by many commands. In practice, the bit index does not need to be known and should not be used directly but through its name to abstract its value.

7 Environments

The object model allows to transform an object into an environment; in other words, a global workspace for a given context, i.e. scope. Objects-as-environments are managed by the methods `open_env`, `close_env`, `load_env`, `dump_env` and `is_open_env`. Things defined in this workspace will be stored in the object, and accessible from outside using the standard ways to access object attributes:

```
local object in MAD
local one = 1
local obj = object { a:=one } -- obj with 'a' defined
-- local a = 1                -- see explication below

obj:open_env()                -- open environment
b = 2                         -- obj.b defined
c =\ -> a..":"..b             -- obj.c defined
obj:close_env()               -- close environment
```

```
print(obj.a, obj.b, obj.c)    -- display: 1   2   1:2
one = 3
print(obj.a, obj.b, obj.c)    -- display: 3   2   3:2
obj.a = 4
print(obj.a, obj.b, obj.c)    -- display: 4   2   4:2
```

Uncommenting the line `local a = 1` would change the last displayed column to `1:2` for the three prints because the *lambda* defined for `obj.c` would capture the local `a` as it would exist in its scope. As seen hereabove, once the environment is closed, the object still holds the variables as attributes.

The MADX environment is an object that relies on this powerful feature to load MAD-X lattices, their settings and their "business logic", and provides functions, constants and elements to mimic the behavior of the global workspace of MAD-X to some extend:

```
MADX:open_env()
mq_k1 = 0.01                      -- mq.k1 is not a valid identifier!
MQ = QUADRUPOLE {l=1, k1:=MQ_K1} -- MADX environment is case insensitive
MADX:close_env()                  -- but not the attributes of objects!
local mq in MADX
print(mq.k1)                      -- display: 0.01
MADX.MQ_K1 = 0.02
print(mq.k1)                      -- display: 0.02
```

Note that MAD-X workspace is case insensitive and everything is "global" (no scope, namespaces), hence the `quadrupole` element has to be directly available inside the MADX environment. Moreover, the MADX object adds the method `load` to extend `load_env` and ease the conversion of MAD-X lattices. For more details see chapter 9.

Chapter 4. Beams

The beam object is the *root object* of beams that store information relative to particles and particle beams. It also provides a simple interface to the particles and nuclei database.

The beam module extends the typeid module with the is_beam *function*, which returns true if its argument is a beam object, false otherwise.

1 Attributes

The beam *object* provides the following attributes:

particle	A *string* specifying the name of the particle. (default: "positron").
mass	A *number* specifying the energy-mass of the particle [GeV]. (default: emass).
charge	A *number* specifying the charge of the particle in [q] unit of qelect.[1] (default: 1).
spin	A *number* specifying the spin of the particle. (default: 0).
emrad	A *lambda* returning the electromagnetic radius of the particle [m], emrad = krad_GeV*charge^2/mass where krad_GeV $= 10^{-9}(4\pi\varepsilon_0)^{-1}q$.
aphot	A *lambda* returning the average number of photon emitted per bending unit, aphot = kpht_GeV*charge^2*betgam where kpht_GeV $= \frac{5}{2\sqrt{3}}$ krad_GeV $(\hbar c)^{-1}$.
energy	A *number* specifying the particle energy [GeV]. (default: 1).
pc	A *lambda* returning the particle momentum times the speed of light [GeV], pc = (energy^2 - mass^2)$^{\frac{1}{2}}$.
beta	A *lambda* returning the particle relativistic $\beta = \frac{v}{c}$, beta = (1 - (mass/energy)^2)$^{\frac{1}{2}}$.
gamma	A *lambda* returning the particle Lorentz factor $\gamma = (1-\beta^2)^{-\frac{1}{2}}$, gamma = energy/mass.
betgam	A *lambda* returning the product $\beta\gamma$, betgam = (gamma^2 - 1)$^{\frac{1}{2}}$.
pc2	A *lambda* returning pc^2, avoiding the square root.
beta2	A *lambda* returning beta^2, avoiding the square root.
betgam2	A *lambda* returning betgam^2, avoiding the square root.
brho	A *lambda* returning the magnetic rigidity [T.m], brho = GeV_c * pc/|charge| where GeV_c $= 10^9/c$.
ex	A *number* specifying the horizontal emittance ϵ_x [m]. (default: 1).
ey	A *number* specifying the vertical emittance ϵ_y [m]. (default: 1).
et	A *number* specifying the longitudinal emittance ϵ_t [m]. (default: 1e-3).

[1]The qelect value is defined in the constant module.

exn A *lambda* returning the normalized horizontal emittance [m],
`exn = ex * betgam`.

eyn A *lambda* returning the normalized vertical emittance [m],
`eyn = ey * betgam`.

etn A *lambda* returning the normalized longitudinal emittance [m],
`etn = et * betgam`.

nbunch A *number* specifying the number of particle bunches in the machine. (default: `0`).

npart A *number* specifying the number of particles per bunch. (default: `0`).

sigt A *number* specifying the bunch length in $c\sigma_t$. (default: `1`).

sige A *number* specifying the relative energy spread in σ_E/E [GeV]. (default: `1e-3`).

The `beam` *object* also implements a special protect-and-update mechanism for its attributes to ensure consistency and precedence between the physical quantities stored internally:

- The following attributes are *read-only*, i.e. writing to them triggers an error:

  `mass, charge, spin, emrad, aphot.`

- The following attributes are *read-write*, i.e. hold values, with their accepted numerical ranges:

  `particle, energy` $>$`mass,`
  `ex` >0, `ey` >0, `et` >0,
  `nbunch` >0, `npart` >0, `sigt` >0, `sige` >0.

- The following attributes are *read-update*, i.e. setting these attributes update the `energy`, with their accepted numerical ranges:

  `pc` >0, $0.9>$ `beta` >0, `gamma` >1, `betgam` >0.1, `brho` >0,
  `pc2, beta2, betgam2.`

- The following attributes are *read-update*, i.e. setting these attributes update the emittances `ex`, `ey`, and `et` repectively, with their accepted numerical ranges:

  `exn` >0, `eyn` >0, `etn` >0.

2 Methods

The `beam` object provides the following methods:

new_particle A *method* `(particle, mass, charge, [spin])` creating new particles or nuclei and store them in the particles database. The arguments specify in order the new `particle`'s name, energy-`mass` [GeV], `charge` [q], and `spin` (default: `0`). These arguments can also be grouped into a *table* with same attribute names as the argument names and passed as the solely argument.

set_variables A *method* `(set)` returning `self` with the attributes set to the pairs (*key*, *value*) contained in `set`. This method overrides the original one to implement the special protect-and-update mechanism, but the order of the updates is undefined. It also creates new particle on-the-fly if the `mass` and the `charge` are defined, and then select it. Shortcut `setvar`.

showdb A *method* `([file])` displaying the content of the particles database to `file` (default: `io.stdout`).

3 Metamethods

The `beam` object provides the following metamethods:

`__init` A *metamethod* `()` returning `self` after having processed the attributes with the special protect-and-update mechanism, where the order of the updates is undefined. It also creates new particle on-the-fly if the `mass` and the `charge` are defined, and then select it.

`__newindex` A *metamethod* `(key, val)` called by the assignment operator `[key]=val` to create new attributes for the pairs (*key*, *value*) or to update the underlying physical quantity of the `beam` objects.

The following attribute is stored with metamethods in the metatable, but has different purpose:

`__beam` A unique private *reference* that characterizes beams.

4 Particles database

The `beam` *object* manages the particles database, which is shared by all `beam` instances. The default set of supported particles is:

```
electron, positron, proton, antiproton, neutron, antineutron, ion, muon,
antimuon, deuteron, antideuteron, negmuon (=muon), posmuon (=antimuon).
```

New particles can be added to the database, either explicitly using the `new_particle` method, or by creating or updating a beam *object* and specifying all the attributes of a particle, i.e. `particle`'s name, `charge`, `mass`, and (optional) `spin`:

```
local beam in MAD
local nmass, pmass, mumass in MAD.constant

-- create a new particle
beam:new_particle{ particle='mymuon', mass=mumass, charge=-1, spin=1/2 }

-- create a new beam and a new nucleus
local pbbeam = beam { particle='pb208', mass=82*pmass+126*nmass, charge=82 }
```

The particles database can be displayed with the `showdb` method at any time from any beam:

```
beam:showdb()  -- check that both, mymuon and pb208 are in the database.
```

5 Particle charges

The physics of MAD-NG is aware of particle charges. To enable the compatibility with codes like MAD-X that ignores the particle charges, the global option `nocharge` can be used to control the behavior of created beams as shown by the following example:

```
local beam, option in MAD
local beam1 = beam { particle="electron" } -- beam with negative charge
print(beam1.charge, option.nocharge)       -- display: -1  false

option.nocharge = true                     -- disable particle charges
local beam2 = beam { particle="electron" } -- beam with negative charge
print(beam2.charge, option.nocharge)       -- display:  1  true
```

```
-- beam1 was created before nocharge activation...
print(beam1.charge, option.nocharge)       -- display: -1  true
```

This approach ensures consistency of beams behavior during their entire lifetime.[2]

6 Examples

The following code snippet creates the LHC lead beams made of bare nuclei $^{208}\mathrm{Pb}^{82+}$:

```
local beam in MAD
local lhcb1, lhcb2 in MADX
local nmass, pmass, amass in MAD.constant
local pbmass = 82*pmass+126*nmass

-- attach a new beam with a new particle to lhcb1 and lhcb2.
lhc1.beam = beam 'Pb208' { particle='pb208', mass=pbmass, charge=82 }
lhc2.beam = lhc1.beam -- let sequences share the same beam...

-- print Pb208 nuclei energy-mass in GeV and unified atomic mass.
print(lhcb1.beam.mass, lhcb1.beam.mass/amass)
```

[2]The option `rbarc` in MAD-X is too volatile and does not ensure such consistency...

Chapter 5. Beta0 Blocks

TODO

The `beta0` object is the *root object* of beta0 blocks that store information relative to the phase space at given positions, e.g. initial conditions, Poincaré section.

The `beta0` module extends the `typeid` module with the `is_beta0` *function*, which returns `true` if its argument is a `beta0` object, `false` otherwise.

1 Attributes

The `beta0` *object* provides the following attributes:

`particle` A *string* specifying the name of the particle. (default: `"positron"`).

2 Methods

The `beta0` object provides the following methods:

`showdb` A *method* `([file])` displaying the content of the particles database to `file` (default: `io.stdout`).

3 Metamethods

The `beta0` object provides the following metamethods:

`__init` A *metamethod* `()` returning `self` after having processed the attributes with the special protect-and-update mechanism, where the order of the updates is undefined. It also creates new particle on-the-fly if the `mass` and the `charge` are defined, and then select it.

The following attribute is stored with metamethods in the metatable, but has different purpose:

`__beta0` A unique private *reference* that characterizes beta0 blocks.

4 Examples

Chapter 6. Elements

The `element` object is the *root object* of all elements used to model particle accelerators, including sequences and drifts. It provides most default values inherited by all elements.

The `element` module extends the `typeid` module with the `is_element` *function*, which returns `true` if its argument is an `element` object, `false` otherwise.

1 Taxonomy

The classes defined by the `element` module are organized according to the kinds and the roles of their instances. The classes defining the kinds are:

`thin` The *thin* elements have zero-length and their physics does not depend on it, i.e. the attribute `l` is discarded or forced to zero in the physics.

`thick` The *thick* elements have a length and their physics depends on it. Elements like `sbend`, `rbend`, `quadrupole`, `solenoid`, and `elseparator` trigger a runtime error if they have zero-length. Other thick elements will accept to have zero-length for compatibility with MAD-X[1], but their physics will have to be adjusted.[2]

`drift` The *drift* elements have a length with a `drift`-like physics if $l \geqslant$ `minlen`[3] otherwise they are discarded or ignored. Any space between elements with a length $l \geqslant$ `minlen` are represented by an *implicit* drift created on need by the s-iterator of sequences and discarded afterward.

`patch` The *patch* elements have zero-length and the purpose of their physics is to change the reference frame.

`extrn` The *extern* elements are never part of sequences. If they are present in a sequence definition, they are expanded and replaced by their content, i.e. stay external to the lattice.

`specl` The *special* elements have special roles like *marking* places (i.e. `maker`) or *branching* sequences (i.e. `slink`).

These classes are not supposed to be used directly, except for extending the hierarchy defined by the `element` module and schematically reproduced hereafter to help users understanding:

```
 thin_element = element  'thin_element' { is_thin    = true }
thick_element = element 'thick_element' { is_thick   = true }
drift_element = element 'drift_element' { is_drift   = true }
patch_element = element 'patch_element' { is_patch   = true }
extrn_element = element 'extrn_element' { is_extern  = true }
specl_element = element 'specl_element' { is_special = true }

sequence    = extrn_element 'sequence'    { }
assembly    = extrn_element 'assembly'    { }
bline       = extrn_element 'bline'       { }
```

[1]In MAD-X, zero-length `sextupole` and `octupole` are valid but may have surprising effects...

[2]E.g. zero-length `sextupole` must define their strength with `knl[3]` instead of `k2` to have the expected effect.

[3]By default `minlen`$= 10^{-12}$m.

```
marker      = specl_element 'marker'      { }
slink       = specl_element 'slink'       { }

drift       = drift_element 'drift'       { }
collimator  = drift_element 'collimator'  { }
instrument  = drift_element 'instrument'  { }
placeholder = drift_element 'placeholder' { }

sbend       = thick_element 'sbend'       { }
rbend       = thick_element 'rbend'       { }
quadrupole  = thick_element 'quadrupole'  { }
sextupole   = thick_element 'sextupole'   { }
octupole    = thick_element 'octupole'    { }
decapole    = thick_element 'decapole'    { }
dodecapole  = thick_element 'dodecapole'  { }
solenoid    = thick_element 'solenoid'    { }
tkicker     = thick_element 'tkicker'     { }
wiggler     = thick_element 'wiggler'     { }
elseparator = thick_element 'elseparator' { }
rfcavity    = thick_element 'rfcavity'    { }
genmap      = thick_element 'genmap'      { }

beambeam    = thin_element  'beambeam'    { }
multipole   = thin_element  'multipole'   { }

xrotation   = patch_element 'xrotation'   { }
yrotation   = patch_element 'yrotation'   { }
srotation   = patch_element 'srotation'   { }
translate   = patch_element 'translate'   { }
changeref   = patch_element 'changeref'   { }
changedir   = patch_element 'changedir'   { }
changenrj   = patch_element 'changenrj'   { }

-- specializations
rfmultipole = rfcavity      'rfmultipole' { }
crabcavity  = rfmultipole   'crabcavity'  { }

monitor     = instrument     'monitor'    { }
hmonitor    = monitor       'hmonitor'    { }
vmonitor    = monitor       'vmonitor'    { }

kicker      = tkicker        'kicker'     { }
hkicker     =  kicker       'hkicker'     { }
vkicker     =  kicker       'vkicker'     { }
```

All the classes above, including `element`, define the attributes `kind = *name*` and `is_*name* = true` where *name* correspond to the class name. These attributes help to identify the kind and the role of an element as shown in the following code excerpt:

```
local drift, hmonitor, sequence in MAD.element
local dft = drift    {}
local bpm = hmonitor {}
local seq = sequence {}
print(dft.kind)              -- display: drift
print(dft.is_drift)          -- display: true
print(dft.is_drift_element)  -- display: true
print(bpm.kind)              -- display: hmonitor
print(bpm.is_hmonitor)       -- display: true
print(bpm.is_monitor)        -- display: true
print(bpm.is_instrument)     -- display: true
print(bpm.is_drift_element)  -- display: true
print(bpm.is_element)        -- display: true
print(bpm.is_drift)          -- display: true
print(bpm.is_thick_element)  -- display: nil (not defined = false)
print(seq.kind)              -- display: sequence
print(seq.is_element)        -- display: true
print(seq.is_extrn_element)  -- display: true
print(seq.is_thick_element)  -- display: nil (not defined = false)
```

2 Attributes

The `element` *object* provides the following attributes:

`l` A *number* specifying the physical length of the element on the design orbit [m]. (default: `0`).

`lrad` A *number* specifying the field length of the element on the design orbit considered by the radiation [m]. (default: `lrad = \s -> s.l`).

`angle` A *number* specifying the bending angle α of the element [rad]. A positive angle represents a bend to the right, i.e. a $-y$-rotation towards negative x values. (default: `0`).

`tilt` A *number* specifying the physical tilt of the element [rad]. All the physical quantities defined by the element are in the tilted frame, except `misalign` that comes first when tracking through an element, see the `track` command for details. (default: `0`).

`model` A *string* specifying the integration model `"DKD"` or `"TKT"` to use when tracking through the element and overriding the command attribute, see the `track` command for details. (default: *`cmd`*`.model`).

`method` A *number* specifying the integration order 2, 4, 6, or 8 to use when tracking through the element and overriding the command attribute, see the `track` command for details. (default: *`cmd`*`.method`).

`nslice` A *number* specifying the number of slices or a *list* of increasing relative positions or a *callable* `(elm, mflw, lw)` returning one of the two previous kind of positions specification to use when tracking through the element and overriding the command attribute, see the `survey` or the `track` commands for details. (default: *`cmd`*`.nslice`).

`refpos` A *string* holding one of `"entry"`, `"centre"` or `"exit"`, or a *number* specifying a position in [m] from the start of the element, all of them resulting in an offset to substract to the `at` attribute to find the s-position of the element entry when inserted in a sequence, see elements positions for details. (default: `nil` $\equiv$ `seq.refer`).

aperture A *mappable* specifying aperture attributes, see Aperture for details. (default: `{kind='circle', 1}`).

apertype A *string* specifying the aperture type, see Aperture for details. (default: `\s -> s.aperture.kind or 'circle'`).[4]

misalign A *mappable* specifying misalignment attributes, see Misalignment for details. (default: `nil`).

The `thick_element` *object* adds the following multipolar and fringe fields attributes:

knl, ksl A *list* specifying respectively the **multipolar** and skew integrated strengths of the element [m^{-i+1}]. (default: `{}`).

dknl, dksl A *list* specifying respectively the multipolar and skew integrated strengths errors of the element [m^{-i+1}]. (default: `{}`).

e1, e2 A *number* specifying respectively the horizontal angle of the pole faces at entry and exit of the element [rad]. A positive angle goes toward inside the element, see Figures 6.1 and 6.2. (default: `0`).

h1, h2 A *number* specifying respectively the horizontal curvature of the pole faces at entry and exit of the element [m^{-1}]. A positive curvature goes toward inside the element. (default: `0`).

hgap A *number* specifying half of the vertical gap at the center of the pole faces of the element [m]. (default: `0`).

fint A *number* specifying the fringe field integral at entrance of the element. (default: `0`).

fintx A *number* specifying the fringe field integral at exit of the element. (default: `fint`).

fringe A *number* specifying the bitmask to activate fringe fields of the element, see Flags for details. (default: `0`).

fringemax A *number* specifying the maximum order for multipolar fringe fields of the element. (default: `2`).

kill_ent_fringe A *logical* specifying to kill the entry fringe fields of the element. (default: `false`).

kill_exi_fringe A *logical* specifying to kill the entry fringe fields of the element. (default: `false`).

f1, f2 A *number* specifying quadrupolar fringe field first and second parameter of SAD. (default: `0`).

3 Methods

The `element` object provides the following methods:

select A *method* `([flg])` to select the element for the flags `flg` (default: `selected`).

deselect A *method* `([flg])` to deselect the element for the flags `flg` (default: `selected`).

is_selected A *method* `([flg])` to test the element for the flags `flg` (default: `selected`).

[4]This attribute was introduced to ease the translation of MAD-X sequences and may disappear in some future.

is_disabled A *method* `()` to test if the element is *disabled*, which is equivalent to call the method `is_selected(disabled)`.

is_observed A *method* `()` to test if the element is *observed*, which is equivalent to call the method `is_selected(observed)`.

is_implicit A *method* `()` to test if the element is *implicit*, which is equivalent to call the method `is_selected(implicit)`.

The `drift_element` and `thick_element` objects provide the following extra methods, see sub-elements for details about the `sat` attribute:

index_sat A *method* `(sat, [cmp])` returning the lowest index `idx` (starting from 1) of the first sub-element with a relative position from the element entry that compares `true` with the *number* `sat` using the optional *callable* `cmp(sat, self[idx].sat)` (default: `"=="`), or `#self+1`. In the presence of multiple equal positions, `"<="` (resp. `">="`) will return the lowest index of the position while `"<"` (resp. `">"`) the lowest index next to the position for ascending (resp. descending) order.

insert_sat A *method* `(elm, [cmp])` returning the element after inserting the sub-element `elm` at the index determined by `:index_sat(elm.sat, [cmp])` using the optional *callable* `cmp` (default: `"<"`).

replace_sat A *method* `(elm)` returning the replaced sub-element found at the index determined by `:index_sat(elm.sat)` by the new sub-element `elm`, or `nil`.

remove_sat A *method* `(sat)` returning the removed sub-element found at the index determined by `:index_sat(sat)`, or `nil`.

4 Metamethods

The `element` object provides the following metamethods:

__len A *metamethod* `()` overloading the length operator `#` to return the number of subelements in the *list* part of the element.

__add A *metamethod* `(obj)` overloading the binary operator `+` to build a `bline` object from the juxtaposition of two elements.

__mul A *metamethod* `(n)` overloading the binary operator `*` to build a `bline` object from the repetition of an element `n` times, i.e. one of the two operands must be a *number*.

__unm A *metamethod* `(n)` overloading the unary operator `-` to build a `bline` object from the turning of an element, i.e. reflect the element.

__tostring A *metamethod* `()` returning a *string* built from the element information, e.g. `print(monitor 'bpm' {})` display the *string* `":monitor: 'bpm' memory-address"`.

The operators overloading of elements allows to unify sequence and beamline definitions in a consistent and simple way, noting that `sequence` and `bline` are (external) elements too.

The following attribute is stored with metamethods in the metatable, but has different purpose:

__elem A unique private *reference* that characterizes elements.

5 Elements

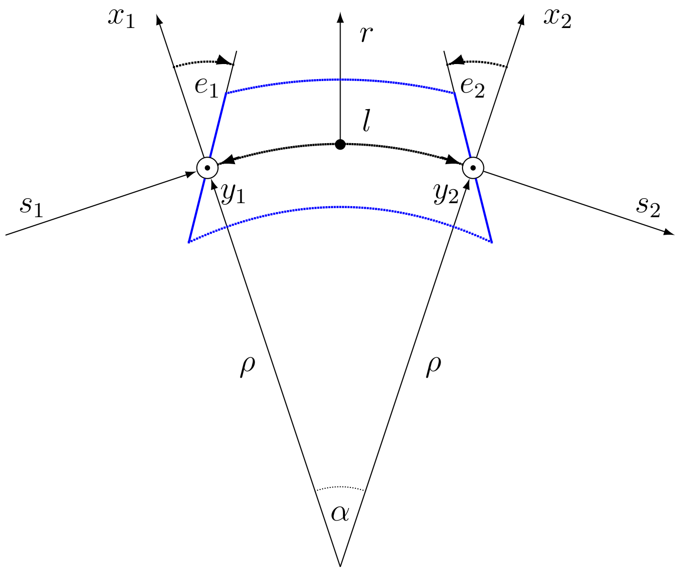
Some elements define new attributes or override the default values provided by the *root object* `element`. The following subsections describe the elements supported by MAD-NG.

5.1 SBend

The `sbend` element is a sector bending magnet with a curved reference system as shown in Figure 6.1, and defines or overrides the following attributes:

k0	A *number* specifying the dipolar strength of the element [m^{-1}]. (default: `k0 = \s -> s.angle/s.l`).[5,6]
k0s	A *number* specifying the dipolar skew strength of the element [m^{-1}]. (default: `0`).
k1, k1s	A *number* specifying respectively the quadrupolar and skew strengths of the element [m^{-2}]. (default: `0`).
k2, k2s	A *number* specifying respectively the sextupolar and skew strengths of the element [m^{-3}]. (default: `0`).
fringe	Set to flag `fringe.bend` to activate the fringe fields by default, see Flags for details.

Figure 6.1: Reference system for a sector bending magnet.



5.2 RBend

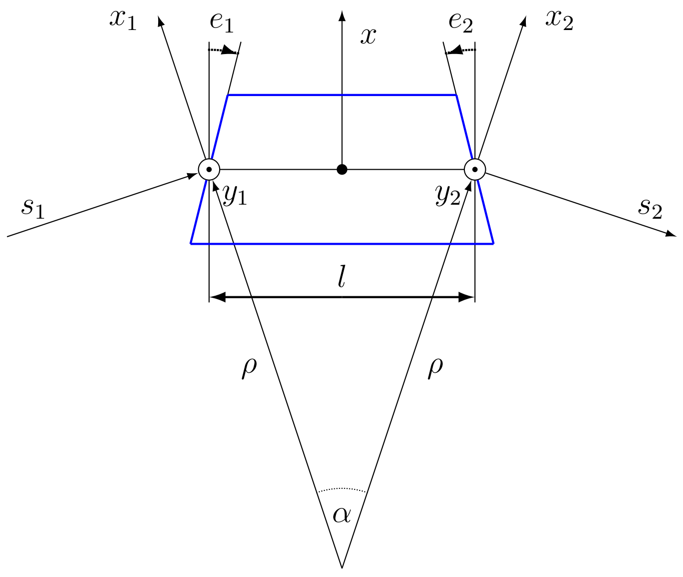
The `rbend` element is a rectangular bending magnet with a straight reference system as shown in Figure 6.2, and defines or overrides the following attributes:

k0	A *number* specifying the dipolar strength of the element [m^{-1}]. (default: `k0 = \s -> s.angle/s.l`).[5,6]

[5]By default bending magnets are ideal bends, that is `angle = k0*l`.
[6]For compatibility with MAD-X.

k0s	A *number* specifying the dipolar skew strength of the element [m^{-1}]. (default: 0).
k1, k1s	A *number* specifying respectively the quadrupolar and skew strengths of the element [m^{-2}]. (default: 0).
k2, k2s	A *number* specifying respectively the sextupolar and skew strengths of the element [m^{-3}]. (default: 0).
fringe	Set to flag `fringe.bend` to activate the fringe fields by default, see Flags for details.
true_rbend	A *logical* specifying if this `rbend` element behaves like (`false`) a `sbend` element with parallel pole faces, i.e. $e_1 = e_2 = \alpha/2$ in Figure 6.1, or like (`true`) a rectangular bending magnet with a straight reference system as shown in Figure 6.2. (default: `false`).[6]

Figure 6.2: Reference system for a rectangular bending magnet.



5.3 Quadrupole

The `quadrupole` element is a straight focusing element and defines the following attributes:

k0, k0s	A *number* specifying respectively the dipolar and skew strengths of the element [m^{-1}]. (default: 0).
k1, k1s	A *number* specifying respectively the quadrupolar and skew strengths of the element [m^{-2}]. (default: 0).
k2, k2s	A *number* specifying respectively the sextupolar and skew strengths of the element [m^{-3}]. (default: 0).

5.4 Sextupole

The `sextupole` element is a straight element and defines the following attributes:

k2, k2s	A *number* specifying respectively the sextupolar and skew strengths of the element [m^{-3}]. (default: 0).

5.5 Octupole

The `octupole` element is a straight element and defines the following attributes:

`k3, k3s`	A *number* specifying respectively the octupolar and skew strengths of the element [m^{-4}]. (default: `0`).

5.6 Decapole

The `decapole` element is a straight element and defines the following attributes:

`k4, k4s`	A *number* specifying respectively the decapolar and skew strength of the element [m^{-5}]. (default: `0`).

5.7 Dodecapole

The `dodecapole` element is a straight element and defines the following attributes:

`k5, k5s`	A *number* specifying respectively the dodecapolar and skew strength of the element [m^{-6}]. (default: `0`).

5.8 Solenoid

The `solenoid` element defines the following attributes:

`ks, ksi`	A *number* specifying respectively the strength [rad/m] and the integrated strength [rad] of the element. A positive value points toward positive s. (default: `0`).

5.9 Multipole

The `multipole` element is a thin element and defines the following attributes:

`knl, ksl`	A *list* specifying respectively the multipolar and skew integrated strengths of the element [m^{-i+1}]. (default: `{}`).
`dknl, dksl`	A *list* specifying respectively the multipolar and skew integrated strengths errors of the element [m^{-i+1}]. (default: `{}`).

5.10 TKicker

The `tkicker` element is the *root object* of kickers and defines or overrides the following attributes:

`hkick`	A *number* specifying the horizontal strength of the element [m^{-1}]. By convention, a kicker with a positive horizontal strength kicks in the direction of the reference orbit, e.g. `hkick` $\equiv$ `-knl[1]`. (default: `0`).
`vkick`	A *number* specifying the vertical strength of the element [m^{-1}]. By convention, a kicker with a positive vertical strength kicks toward the reference orbit, e.g. `vkick` $\equiv$ `ksl[1]`. (default: `0`).
`method`	Set to `2` if `ptcmodel` is not set to enforce pure momentum kick and avoid dipolar strength integration that would introduce dispersion.

5.11 Kicker, HKicker, VKicker

The `kicker` element inheriting from the `tkicker` element, is the *root object* of kickers involved in the orbit correction and defines the following attributes:

`chkick, cvkick` A *number* specifying respectively the horizontal and vertical correction strength of the element set by the `correct` command [m^{-1}]. (default: `0`).

The `hkicker` (horizontal kicker) and `vkicker` (vertical kicker) elements define the following attribute:

`kick` A *number* specifying the strength of the element in its main direction [m^{-1}]. (default: `0`).

5.12 Monitor, HMonitor, VMonitor

The `monitor` element is the root object of monitors involved in the orbit correction and defines the following attributes:

`mredx, mredy` A *number* specifying respectively the readout x,y-offset error of the element [m]. The offset is added to the beam position during orbit correction (after scaling). (default: `0`).

`mresx, mresy` A *number* specifying respectively the readout x,y-scaling error of the element. The scale factor multiplies the beam position by `1+mres` (before offset) during orbit correction.[7] (default: `0`).

The `hmonitor` (horizontal monitor) and `vmonitor` (vertical monitor) elements are specialisations inheriting from the `monitor` element.

5.13 RFCavity

The `rfcavity` element defines the following attributes:

`volt` A *number* specifying the peak RF voltage of the element [MV]. (default: `0`).

`freq` A *number* specifying a non-zero RF frequency of the element [MHz]. (default: `0`).

`lag` A *number* specifying the RF phase lag of the element in unit of 2π. (default: `0`).

`harmon` A *number* specifying the harmonic number of the element if `freq` is zero. (default: `0`).

`n_bessel` A *number* specifying the transverse focussing effects order of the element. (default: `0`).

`totalpath` A *logical* specifying if the totalpath must be used in the element. (default: `true`).

5.14 RFMultipole

The `rfmultipole` element defines the following attributes:

`pnl, psl` A *list* specifying respectively the multipolar and skew phases of the element [rad]. (default: `{}`).

`dpnl, dpsl` A *list* specifying respectively the multipolar and skew phases errors of the element [rad]. (default: `{}`).

[7] This definition comes from MAD-X default zeroed values such that undefined attribute gives a scale of `1`.

5.15 ElSeparator

The `elseparator` element defines the following attributes:

`ex, ey` A *number* specifying respectively the electric field x,y-strength of the element [MV/m]. (default: `0`).

`exl, eyl` A *number* specifying respectively the integrated electric field x,y-strength of the element [MV]. (default: `0`).

5.16 Wiggler

The `wiggler` element defines the following attributes: NYI, TBD

5.17 BeamBeam

The `beambeam` element defines the following attributes: NYI, TBD

5.18 GenMap

The `genmap` element defines the following attributes:[8]

`damap` A *damap* used for thick integration.

`update` A *callable* `(elm, mflw, lw)` invoked before each step of thick integration to update the *damap*. (default: `nil`)

`nslice` A *number* specifying the number of slices or a *list* of increasing relative positions or a *callable* `(elm, mflw, lw)` returning one of the two previous kind of positions specification to use when tracking through the element and overriding the command attribute, see the `survey` or the `track` commands for details. (default: `1`).

5.19 SLink

The `slink` element defines the following attributes:[9]

`sequence` A *sequence* to switch to right after exiting the element. (default: `nil`)

`range` A *range* specifying the span over the sequence to switch to, as expected by the sequence method `:siter`. (default: `nil`).

`nturn` A *number* specifying the number of turn to track the sequence to switch to, as expected by the sequence method `:siter`. (default: `nil`).

`dir` A *number* specifying the s-direction of the tracking of the sequence to switch to, as expected by the sequence method `:siter`. (default: `nil`).

`update` A *callable* `(elm, mflw)` invoked before retrieving the other attributes when entering the element. (default: `nil`)

[8] This element is a generalization of the `matrix` element of MAD-X, to use with care!

[9] This element allows to switch between sequences during tracking, kind of `if-then-else` for tracking.

5.20 Translate

The `translate` element is a patch element and defines the following attributes:

`dx, dy, ds` A *number* specifying respectively x,y,s-translation of the reference frame [m]. (default: `0`)

5.21 XRotation, YRotation, SRotation

The `xrotation` (rotation around x-axis), `yrotation` (rotation around y-axis) and `srotation` (rotation around s-axis) elements are patches element and define the following attribute:

`angle` A *number* specifying the rotation angle around the axis of the element [rad]. (default: `0`).

5.22 ChangeRef

The `changeref` element is a patch element and defines the following attributes:

`dx, dy, ds` A *number* specifying respectively x,y,s-translation of the reference frame [m]. (default: `0`)

`dtheta, dphi, dpsi` A *number* specifying respectively y,$-x$,s-rotation of the reference frame applied in this order after any translation [rad]. (default: `0`)

5.23 ChangeDir

The `changedir` element is a patch element that reverses the direction of the sequence during the tracking.

5.24 ChangeNrj

The `changenrj` element is a patch element and defines the following attributes:

`dnrj` A *number* specifying the change by δ_E of the *reference* beam energy [GeV]. The momenta of the particles or damaps belonging to the reference beam (i.e. not owning a beam) are updated, while other particles or damaps owning their beam are ignored. (default: `0`)

6 Flags

The `element` module exposes the following *object* flags through `MAD.element.flags` to use in conjunction with the methods `select` and `deselect`:[10]

`none` All bits zero.

`selected` Set if the element has been selected.

`disabled` Set if the element has been disabled, e.g. for orbit correction.

`observed` Set if the element has been selected for observation, e.g. for output to TFS table. The `$end` markers are selected for observation by default, and commands with the `observe` attribute set to `0` discard this flag and consider all elements as selected for observation.

[10]Remember that flags are *not* inherited nor copied as they are qualifying the object itself.

implicit Set if the element is implicit, like the temporary *implicit* drifts created on-the-fly by the `sequence` s-iterator with indexes at half integers. This flag is used by commands with the `implicit` attribute.

playout Set if the element `angle` must be used by layout plot. This flag is useful to plot multiple sequence layouts around interaction points, like `lhcb1` and `lhcb2` around `IP1` and `IP5`.

7 Fringe fields

The `element` module exposes the following flags through `MAD.element.flags.fringe` to *control* the elements fringe fields through their attribute `fringe`, or to *restrict* the activated fringe fields with the commands attribute `fringe`:[11]

none All bits zero.

bend Control the element fringe fields for bending fields.

mult Control the element fringe fields for multipolar fields up to `fringemax` order.

rfcav Control the element fringe fields for rfcavity fields.

qsad Control the element fringe fields for multipolar fields with extra terms for quadrupolar fields for compatibility with SAD.

comb Control the element fringe fields for combined bending and multipolar fields.

combqs Control the element fringe fields for combined bending and multipolar fields with extra terms for quadrupolar fields for compatibility with SAD.

The *element* `thick_element` provides a dozen of attributes to parametrize the aforementionned fringe fields. Note that in some future, part of these attributes may be grouped into a *mappable* to ensure a better consistency of their parametrization.

8 Sub-elements

An element can have thin or thick sub-elements stored in its *list* part, hence the length operator `#` returns the number of them. The attribute `sat` of sub-elements, i.e. read `sub-at`, is interpreted as their relative position from the entry of their enclosing main element, that is a fractional of its length. The positions of the sub-elements can be made absolute by dividing their `sat` attribute by the length of their main element using lambda expressions. The sub-elements are only considered and valid in the `drift_element` and `thick_element` kinds that implement the methods `:index_sat`, `:insert_sat`, `:remove_sat`, and `:replace_sat` to manage sub-elements from their `sat` attribute. The sequence method `:install` updates the `sat` attribute of the elements installed as sub-elements if the *logical* `elements.subelem` of the packed form is enabled, i.e. when the s-position determined by the `at`, `from` and `refpos` attributes falls inside a non-zero length element already installed in the sequence that is not an *implicit* drift. The physics of thick sub-elements will shield the physics of their enclosing main element along their length, unless they combine their attributes with those of their main element using lambda expressions to select some combined function physics.

[11]Those flags are *not* object flags, but fringe fields flags.

9 Aperture

All the apertures are *mappable* defined by the following attributes in the tilted frame of an element, see the track command for details:

`kind`	A *string* specifying the aperture shape. (no default).
`tilt`	A *number* specifying the tilt angle of the aperture [rad]. (default: `0`).
`xoff, yoff`	A *number* specifying the transverse x, y-offset of the aperture [m]. (default: `0`).
`maper`	A *mappable* specifying a smaller aperture[12] than the `polygon` aperture to use before checking the polygon itself to speed up the test. The attributes `tilt`, `xoff` and `yoff` are ignored and superseded by the ones of the `polygon` aperture. (default: `nil`).

The supported aperture shapes are listed hereafter. The parameters defining the shapes are expected to be in the *list* part of the apertures and defines the top-right sector shape, except for the `polygon`:

`square`	A square shape with one parameter defining the side half-length. It is the default aperture check with limits set to `1`.
`rectangle`	A rectangular shape with two parameters defining the x,y-half lengths (default: `1` [m]).
`circle`	A circular shape with one parameter defining the radius.
`ellipse`	A elliptical shape with two parameters defining the x,y-radii. (default: `1` [m]).
`rectcircle`	A rectangular shape intersected with a circular shape with three parameters defining the x,y-half lengths and the radius. (default: `1` [m]).
`rectellipse`	A rectangular shape intersected with an elliptical shape with four parameters defining the x,y-half lengths and the x,y-radii.
`racetrack`	A rectangular shape with corners rounded by an elliptical shape with four parameters defining the x,y-half lengths and the corners x,y-radii.
`octagon`	A rectangular shape with corners truncated by a triangular shape with four parameters defining the x,y-half lengths and the triangle x,y-side lengths. An octagon can model hexagon or diamond shapes by equating the triangle lengths to the rectangle half-lengths.
`polygon`	A polygonal shape defined by two vectors `vx` and `vy` holding the vertices coordinates. The polygon does not need to be convex, simple or closed, but in the latter case it will be closed automatically by joining the first and the last vertices.
`bbox`	A 6D bounding box with six parameters defining the upper limits of the absolute values of the six coordinates.

The following example defines new classes with three different aperture definitions:

```
local quadrupole in MAD.element
local mq = quadrupole 'mq' { l=1,                          -- new class
  aperture = { kind='racetrack',
               tilt=pi/2, xoff=1e-3, yoff=5e-4,            -- attributes
               0.06,0.06,0.01,0.01 }                       -- parameters
}
```

[12]It is the responsibility of the user to ensure that `maper` defines a smaller aperture than the polygon aperture.

```
local mqdiam = quadrupole 'mqdiam' { l=1,                       -- new class
  aperture = { kind='octogon', xoff=1e-3, yoff=1e-3,            -- attributes
               0.06,0.04,0.06,0.04 }                            -- parameters
}
local mqpoly = quadrupole 'mqpoly' { l=1,                       -- new class
  aperture = { kind='polygon', tilt=pi/2, xoff=1e-3, yoff=1e-3, -- attributes
               vx=vector{0.05, ...}, vy=vector{0, ...},         -- parameters
               aper={kind='circle', 0.05}                       -- 2nd aperture
}
```

10 Misalignment

The misalignments are *mappable* defined at the entry of an element by the following attributes, see the track command for details:

dx, dy, ds A *number* specifying the x,y,s-displacement at the element entry [m], see Figures 6.3 and 6.4. (default: 0).

dtheta A *number* specifying the y-rotation angle (azimuthal) at the element entry [rad], see Figure 6.3. (default: 0).

dphi A *number* specifying the $-x$-rotation angle (elevation) at the entry of the element [rad], see Figure 6.4. (default: 0).

dpsi A *number* specifying the s-rotation angle (roll) at the element entry [rad], see Figure 6.5. (default: 0).

Two kinds of misalignments are available for an element and summed beforehand:

- The *absolute* misalignments of the element versus its local reference frame, and specified by its `misalign` attribute. These misalignments are always considered.
- The *relative* misalignments of the element versus a given sequence, and specified by the *method* `:misalign` of `sequence`. These misalignments can be considered or not depending of command settings.

Figure 6.3: Displacements in the (x, s) plane.

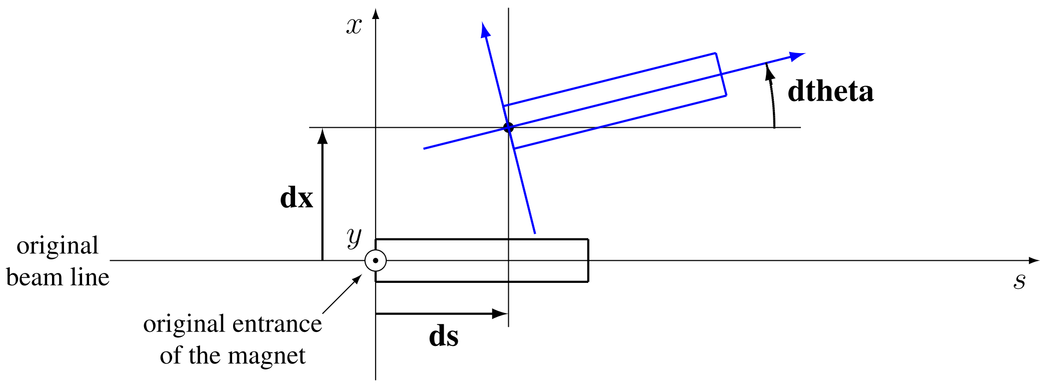


Figure 6.4: Displacements in the (y, s) plane.

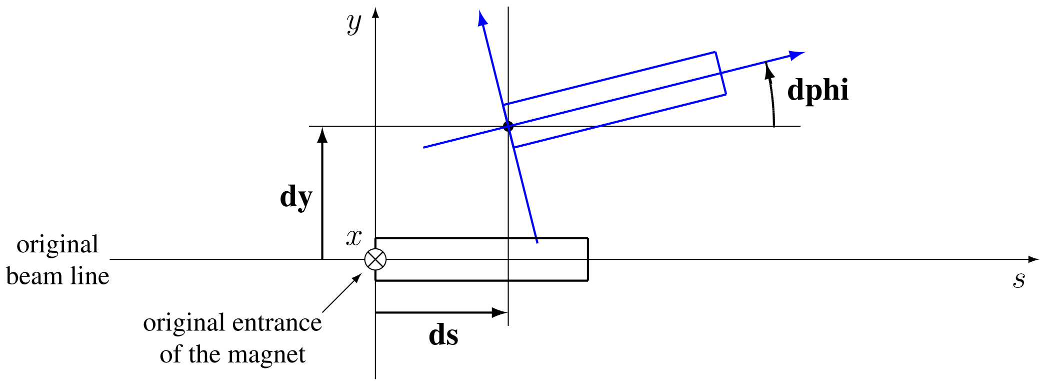
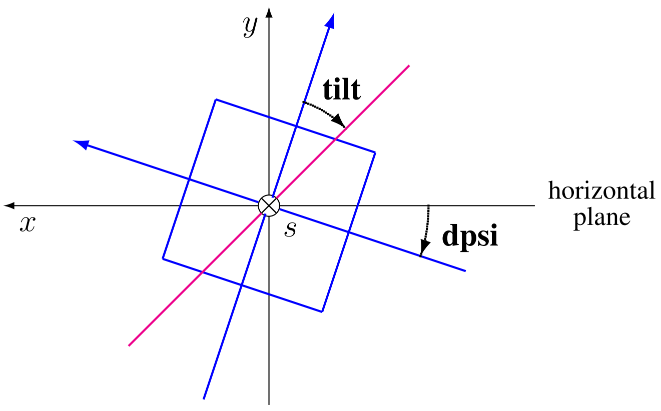


Figure 6.5: Displacements in the (x, y) plane.



Chapter 7. Sequences

The MAD Sequences are objects convenient to describe accelerators lattices built from a *list* of elements with increasing s-positions. The sequences are also containers that provide fast access to their elements by referring to their indexes, s-positions, or (mangled) names, or by running iterators constrained with ranges and predicates.

The `sequence` object is the *root object* of sequences that store information relative to lattices.

The `sequence` module extends the `typeid` module with the `is_sequence` function, which returns `true` if its argument is a `sequence` object, `false` otherwise.

1 Attributes

The `sequence` object provides the following attributes:

`l`	A *number* specifying the length of the sequence [m]. A `nil` will be replaced by the computed lattice length. A value greater or equal to the computed lattice length will be used to place the `$end` marker. Other values will raise an error. (default: `nil`).
`dir`	A *number* holding one of `1` (forward) or `-1` (backward) and specifying the direction of the sequence.[1] (default: `1`)
`refer`	A *string* holding one of `"entry"`, `"centre"` or `"exit"` to specify the default reference position in the elements to use for their placement. An element can override it with its `refpos` attribute, see element positions for details. (default: `nil` ≡ `"centre"`).
`owner`	A *logical* specifying if an *empty* sequence is a view with no data (`owner ~= true`), or a sequence holding data (`owner == true`). (default: `nil`)
`minlen`	A *number* specifying the minimal length [m] to generate *implicit* drifts between elements in s-iterators generated by the method `:siter`. This attribute is automatically set to 10^{-6} m when a sequence is created within the MADX environment. (default: `nil`)
`beam`	An attached `beam`. (default: `nil`)

Warning: the following private and read-only attributes are present in all sequences and should *never be used, set or changed*; breaking this rule would lead to an *undefined behavior*:

`__dat`	A *table* containing all the private data of sequences.
`__cycle`	A *reference* to the element registered with the `:cycle` method. (default: `nil`)

2 Methods

The `sequence` object provides the following methods:

`elem`	A *method* `(idx)` returning the element stored at the positive index `idx` in the sequence, or `nil`.
`spos`	A *method* `(idx)` returning the s-position at the entry of the element stored at the positive index `idx` in the sequence, or `nil`.

[1]This is equivalent to the MAD-X `bv` flag.

`upos`	A *method* `(idx)` returning the s-position at the user-defined `refpos` offset of the element stored at the positive index `idx` in the sequence, or `nil`.
`ds`	A *method* `(idx)` returning the length of the element stored at the positive index `idx` in the sequence, or `nil`.
`align`	A *method* `(idx)` returning a *set* specifying the misalignment of the element stored at the positive index `idx` in the sequence, or `nil`.
`index`	A *method* `(idx)` returning a positive index, or `nil`. If `idx` is negative, it is reflected versus the size of the sequence, e.g. `-1` becomes `#self`, the index of the `$end` marker.
`name_of`	A *method* `(idx, [ref])` returning a *string* corresponding to the (mangled) name of the element at the index `idx` or `nil`. An element name appearing more than once in the sequence will be mangled with an absolute count, e.g. `mq[3]`, or a relative count versus the optional reference element `ref` determined by `:index_of`, e.g. `mq{-2}`.
`index_of`	A *method* `(a, [ref], [dir])` returning a *number* corresponding to the positive index of the element determined by the first argument or `nil`. If `a` is a *number* (or a *string* representing a *number*), it is interpreted as the s-position of an element and returned as a second *number*. If `a` is a *string*, it is interpreted as the (mangled) name of an element as returned by `:name_of`. Finally, `a` can be a *reference* to an element to search for. The argument `ref` (default: `nil`) specifies the reference element determined by `:index_of(ref)` to use for relative s-positions, for decoding mangled names with relative counts, or as the element to start searching from. The argument `dir` (default: `1`) specifies the direction of the search with values `1` (forward), `-1` (backward), or `0` (no direction). The `dir=0` case may return an index at half-integer if `a` is interpreted as an s-position pointing to an *implicit drift*.
`range_of`	A *method* `([rng], [ref], [dir])` returning three *number*s corresponding to the positive indexes *start* and *end* of the range and its direction *dir*, or `nil` for an empty range. If `rng` is omitted, it returns `1`, `#self`, `1`, or `#self`, `1`, `-1` if `dir` is negative. If `rng` is a *number* or a *string* with no `'/'` separator, it is interpreted as both *start* and *end* and determined by `index_of`. If `rng` is a *string* containing the separator `'/'`, it is split in two *string*s interpreted as *start* and *end*, both determined by `:index_of`. If `rng` is a *list*, it will be interpreted as { *start*, *end*, `[ref]`, `[dir]` }, both determined by `:index_of`, unless `ref` equals `'idx'` then both are determined by `:index` (i.e. a *number* is interpreted as an index instead of a s-position). The arguments `ref` (default: `nil`) and `dir` (default: `1`) are forwarded to all invocations of `:index_of` with a higher precedence than ones in the *list* `rng`, and a runtime error is raised if the method returns `nil`, i.e. to disambiguate between a valid empty range and an invalid range.
`length_of`	A *method* `([rng], [ntrn], [dir])` returning a *number* specifying the length of the range optionally including `ntrn` extra turns (default: `0`), and calculated from the indexes returned by `:range_of([rng], nil, [dir])`.
`iter`	A *method* `([rng], [ntrn], [dir])` returning an iterator over the sequence elements. The optional range is determined by `:range_of(rng, [dir])`, optionally including `ntrn` turns (default: `0`). The optional direction `dir` specifies the forward `1` or the backward `-1` direction of the iterator. If `rng` is not provided and the mtable is cycled, the *start* and *end* indexes are determined by `:index_of(self.__cycle)`. When used with a generic **`for`** loop, the iterator returns at each element: its index, the element itself, its s-position over the running loop and its signed length depending on the direction.

`siter`	A *method* `([rng], [ntrn], [dir])` returning an s-iterator over the sequence elements. The optional range is determined by `:range_of([rng], nil, [dir])`, optionally including `ntrn` turns (default: `0`). The optional direction `dir` specifies the forward `1` or the backward `-1` direction of the iterator. When used with a generic **`for`** loop, the iterator returns at each iteration: its index, the element itself or an *implicit* `drift`, its s-position over the running loop and its signed length depending on the direction. Each *implicit* drift is built on-the-fly by the iterator with a length equal to the gap between the elements surrounding it and a half-integer index equal to the average of their indexes. The length of *implicit* drifts is bounded by the maximum between the sequence attribute `minlen` and the `minlen` from the constant module.
`foreach`	A *method* `(act, [rng], [sel], [not])` returning the sequence itself after applying the action `act` on the selected elements. If `act` is a *set* representing the arguments in the packed form, the missing arguments will be extracted from the attributes `action`, `range`, `select` and `default`. The action `act` must be a *callable* `(elm, idx, [midx])` applied to an element passed as first argument and its index as second argument, the optional third argument being the index of the main element in case `elm` is a sub-element. The optional range is used to generate the loop iterator `:iter([rng])`. The optional selector `sel` is a *callable* `(elm, idx, [midx])` predicate selecting eligible elements for the action using the same arguments. The selector `sel` can be specified in other ways, see element selections for details. The optional *logical* `not` (default: `false`) indicates how to interpret default selection, as *all* or *none*, depending on the semantic of the action.[2]
`select`	A *method* `([flg], [rng], [sel], [not])` returning the sequence itself after applying the action `:select([flg])` to the elements using `:foreach(act, [rng], [sel], [not])`. By default sequence have all their elements deselected with only the `$end` marker `observed`.
`deselect`	A *method* `([flg], [rng], [sel], [not])` returning the sequence itself after applying the action `:deselect([flg])` to the elements using `:foreach(act, [rng], [sel], [not])`. By default sequence have all their elements deselected with only the `$end` marker `observed`.
`filter`	A *method* `([rng], [sel], [not])` returning a *list* containing the positive indexes of the elements determined by `:foreach(filt_act, [rng], [sel], [not])`, and its size. The *logical* `sel.subelem` specifies to select sub-elements too, and the *list* may contain non-integer indexes encoding their main element index added to their relative position, i.e. `midx.sat`. The builtin *function* `math.modf(num)` allows to retrieve easily the main element `midx` and the sub-element `sat`, e.g. `midx,sat = math.modf(val)`.
`install`	A *method* `(elm, [rng], [sel], [cmp])` returning the sequence itself after installing the elements in the *list* `elm` at their element positions; unless `from="selected"` is defined meaning multiple installations at positions relative to each element determined by the method `:filter([rng], [sel], true)`. The *logical* `sel.subelem` is ignored. If the arguments are passed in the packed form, the extra attribute `elements` will be used as a replacement for the argument `elm`. The *logical* `elm.subelem` specifies to install elements with s-position falling inside sequence elements as sub-elements, and set their `sat` attribute accordingly. The optional *callable* `cmp(elmspos, spos[idx])` (default: `"<"`) is used to search for the s-position of the installation, where equal s-position are installed after (i.e. before with `"<="`), see `bsearch` from the utility module for details. The *implicit* drifts are checked after each element installation.

[2]For example, the `:remove` method needs `not=true` to *not* remove all elements if no selector is provided.

`replace` A *method* `(elm, [rng], [sel])` returning the *list* of replaced elements by the elements in the *list* `elm` placed at their element positions, and the *list* of their respective indexes, both determined by `:filter([rng], [sel], true)`. The *list* `elm` cannot contain instances of `sequence` or `bline` elements and will be recycled as many times as needed to replace all selected elements. If the arguments are passed in the packed form, the extra attribute `elements` will be used as a replacement for the argument `elm`. The *logical* `sel.subelem` specifies to replace selected sub-elements too and set their `sat` attribute to the same value. The *implicit* drifts are checked only once all elements have been replaced.

`remove` A *method* `([rng], [sel])` returning the *list* of removed elements and the *list* of their respective indexes, both determined by `:filter([rng], [sel], true)`. The *logical* `sel.subelem` specifies to remove selected sub-elements too.

`move` A *method* `([rng], [sel])` returning the sequence itself after updating the element positions at the indexes determined by `:filter([rng], [sel], true)`. The *logical* `sel.subelem` is ignored. The elements must keep their order in the sequence and surrounding *implicit* drifts are checked only once all elements have been moved.[3]

`misalign` A *method* `(algn, [rng], [sel])` returning the sequence itself after setting the element misalignments from `algn` at the indexes determined by `:filter([rng], [sel], true)`. If `algn` is a *mappable*, it will be used to misalign the filtered elements. If `algn` is a *iterable*, it will be accessed using the filtered elements indexes to retrieve their specific misalignment. If `algn` is a *callable* `(idx)`, it will be invoked for each filtered element with their index as solely argument to retrieve their specific misalignment.

`reflect` A *method* `([name])` returning a new sequence from the sequence reversed, and named from the optional *string* `name` (default: `self.name..'_rev'`).

`cycle` A *method* `(a)` returning the sequence itself after checking that `a` is a valid reference using `:index_of(a)`, and storing it in the `__cycle` attribute, itself erased by the methods editing the sequence like `:install`, `:replace`, `:remove`, `:share`, and `:unique`.

`share` A *method* `(seq2)` returning the *list* of elements removed from the `seq2` and the *list* of their respective indexes, and replaced by the elements from the sequence with the same name when they are unique in both sequences.

`unique` A *method* `([fmt])` returning the sequence itself after replacing all non-unique elements by new instances sharing the same parents. The optional `fmt` must be a *callable* `(name, cnt, idx)` that returns the mangled name of the new instance build from the element `name`, its count `cnt` and its index `idx` in the sequence. If the optional `fmt` is a *string*, the mangling *callable* is built by binding `fmt` as first argument to the function `string.format` from the standard library, see Lua 5.2 §6.4 for details.

`publish` A *method* `(env, [keep])` returning the sequence after publishing all its elements in the environment `env`. If the *logical* `keep` is `true`, the method will preserve existing elements from being overridden. This method is automatically invoked with `keep=true` when sequences are created within the `MADX` environment.

`copy` A *method* `([name], [owner])` returning a new sequence from a copy of `self`, with the optional `name` and the optional attribute `owner` set. If the sequence is a view, so will be the copy unless `owner == true`.

[3]Updating directly the positions attributes of an element has no effect.

is_view A *method* `()` returning `true` if the sequence is a view over another sequence data, `false` otherwise.

set_readonly Set the sequence as read-only, including its columns.

save_flags A *method* `([flgs])` saving the flags of all the elements to the optional *iterable* `flgs` (default: `{}`) and return it.

restore_flags A *method* `(flgs)` restoring the flags of all the elements from the *iterable* `flgs`. The indexes of the flags must match the indexes of the elements in the sequence.

dumpseq A *method* `([fil], [info])` displaying on the optional file `fil` (default: `io.stdout`) information related to the position and length of the elements. Useful to identify negative drifts and badly positioned elements. The optional argument `info` indicates to display extra information like elements misalignments.

check_sequ A *method* `()` checking the integrity of the sequence and its dictionary, for debugging purpose only.

3 Metamethods

The `sequence` object provides the following metamethods:

__len A *metamethod* `()` called by the length operator `#` to return the size of the sequence, i.e. the number of elements stored including the `"$start"` and `"$end"` markers.

__index A *metamethod* `(key)` called by the indexing operator `[key]` to return the *value* of an attribute determined by *key*. The *key* is interpreted differently depending on its type with the following precedence:

1. A *number* is interpreted as an element index and returns the element or `nil`.
2. Other *key* types are interpreted as *object* attributes subject to object model lookup.
3. If the *value* associated with *key* is `nil`, then *key* is interpreted as an element name and returns either the element or an *iterable* on the elements with the same name.[4]
4. Otherwise returns `nil`.

__newindex A *metamethod* `(key, val)` called by the assignment operator `[key]=val` to create new attributes for the pairs (*key*, *value*). If *key* is a *number* specifying the index or a *string* specifying the name of an existing element, the following error is raised:

```
"invalid sequence write access (use replace method)"
```

__init A *metamethod* `()` called by the constructor to compute the elements positions.[5]

__copy A *metamethod* `()` similar to the `:copy` *method*.

The following attribute is stored with metamethods in the metatable, but has different purpose:

__sequ A unique private *reference* that characterizes sequences.

[4]An *iterable* supports the length operator `#`, the indexing operator `[]`, and generic `for` loops with `ipairs`.
[5]MAD-NG does not have a MAD-X like `"USE"` command to finalize this computation.

4 Sequences creation

During its creation as an *object*, a sequence can defined its attributes as any object, and the *list* of its elements that must form a *sequence* of increasing s-positions. When subsequences are part of this *list*, they are replaced by their respective elements as a sequence *element* cannot be present inside other sequences. If the length of the sequence is not provided, it will be computed and set automatically. During their creation, sequences compute the s-positions of their elements as described in the section element positions, and check for overlapping elements that would raise a “negative drift” runtime error.

The following example shows how to create a sequence form a *list* of elements and subsequences:

```
local sequence, drift, marker in MAD.element
local df, mk = drift 'df' {l=1}, marker 'mk' {}
local seq = sequence 'seq' {
  df 'df1' {}, mk 'mk1' {},
  sequence {
    sequence { mk 'mk0' {} },
    df 'df.s' {}, mk 'mk.s' {}
  },
  df 'df2' {}, mk 'mk2' {},
} :dumpseq()

-- display
sequence: seq, l=3
idx  kind     name     l          dl       spos       upos     uds
001  marker   $start   0.000       0       0.000      0.000    0.000
002  drift    df1      1.000       0       0.000      0.500    0.500
003  marker   mk1      0.000       0       1.000      1.000    0.000
004  marker   mk0      0.000       0       1.000      1.000    0.000
005  drift    df.s     1.000       0       1.000      1.500    0.500
006  marker   mk.s     0.000       0       2.000      2.000    0.000
007  drift    df2      1.000       0       2.000      2.500    0.500
008  marker   mk2      0.000       0       3.000      3.000    0.000
009  marker   $end     0.000       0       3.000      3.000    0.000
```

5 Elements positions

A sequence looks at the following attributes of an element, including sub-sequences, when installing it, *and only at that time*, to determine its position:

`at`	A *number* holding the position in [m] of the element in the sequence relative to the position specified by the `from` attribute.
`from`	A *string* holding one of `"start"`, `"prev"`, `"next"`, `"end"` or `"selected"`, or the (mangled) name of another element to use as the reference position, or a *number* holding a position in [m] from the start of the sequence. (default: `"start"` if `at`$\geqslant 0$, `"end"` if `at`< 0, and `"prev"` otherwise)
`refpos`	A *string* holding one of `"entry"`, `"centre"` or `"exit"`, or the (mangled) name of a sequence sub-element to use as the reference position, or a *number* specifying a position [m] from the start of the element, all of them resulting in an offset to substract to the `at` attribute to find the s-position of the element entry. (default: `nil` $\equiv$ `self.refer`).

shared A *logical* specifying if an element is used at different positions in the same sequence definition, i.e. shared multiple times, through temporary instances to store the many `at` and `from` attributes needed to specify its positions. Once built, the sequence will drop these temporary instances in favor of their common parent, i.e. the original shared element.

Warning: The `at` and `from` attributes are not considered as intrinsic properties of the elements and are used only once during installation. Any reuse of these attributes is the responsibility of the user, including the consistency between `at` and `from` after updates.

6 Elements selections

The element selection in sequence use predicates in combination with iterators. The sequence iterator manages the range of elements where to apply the selection, while the predicate says if an element in this range is illegible for the selection. In order to ease the use of methods based on the `:foreach` method, the selector predicate `sel` can be built from different types of information provided in a *set* with the following attributes:

flag A *number* interpreted as a flags mask to pass to the element method `:is_selected`. It should not be confused with the flags passed as argument to methods `:select` and `:deselect`, as both flags can be used together but with different meanings!

pattern A *string* interpreted as a pattern to match the element name using `string.match` from the standard library, see Lua 5.2 §6.4 for details.

class An *element* interpreted as a *class* to pass to the element method `:is_instansceOf`.

list An *iterable* interpreted as a *list* used to build a *set* and select the elements by their name, i.e. the built predicate will use `tbl[elm.name]` as a *logical*. If the *iterable* is a single item, e.g. a *string*, it will be converted first to a *list*.

table A *mappable* interpreted as a *set* used to select the elements by their name, i.e. the built predicate will use `tbl[elm.name]` as a *logical*. If the *mappable* contains a *list* or is a single item, it will be converted first to a *list* and its *set* part will be discarded.

select A *callable* interpreted as the selector itself, which allows to build any kind of predicate or to complete the restrictions already built above.

subelem A *boolean* indicating to include or not the sub-elements in the scanning loop. The predicate and the action receive the sub-element and its sub-index as first and second argument, and the main element index as third argument.

All these attributes are used in the aforementioned order to incrementally build predicates that are combined with logical conjunctions, i.e. `and`'ed, to give the final predicate used by the `:foreach` method. If only one of these attributes is needed, it is possible to pass it directly in `sel`, not as an attribute in a *set*, and its type will be used to determine the kind of predicate to build. For example, `self:foreach(act, monitor)` is equivalent to `self:foreach{action=act, class=monitor}`.

7 Indexes, names and counts

Indexing a sequence triggers a complex look up mechanism where the arguments will be interpreted in various ways as described in the `:__index` metamethod. A *number* will be interpreted as a relative slot index in the list of elements, and a negative index will be considered as relative to the end of the sequence, i.e. `-1` is the `$end` marker. Non-*number* will be interpreted first as an object key (can be anything), looking for sequence methods or attributes; then as an element name if nothing was found.

If an element exists but its name is not unique in the sequence, an *iterable* is returned. An *iterable* supports the length `#` operator to retrieve the number of elements with the same name, the indexing operator `[]` waiting for a count n to retrieve the n-th element from the start with that name, and the iterator `ipairs` to use with generic `for` loops.

The returned *iterable* is in practice a proxy, i.e. a fake intermediate object that emulates the expected behavior, and any attempt to access the proxy in another manner should raise a runtime error.

Warning: The indexing operator `[]` interprets a *number* as a (relative) element index as the method `:index`, while the method `:index_of` interprets a *number* as a (relative) element s-position [m].

The following example shows how to access to the elements through indexing and the *iterable*:

```
local sequence, drift, marker in MAD.element
local seq = sequence {
  drift 'df' { id=1 }, marker 'mk' { id=2 },
  drift 'df' { id=3 }, marker 'mk' { id=4 },
  drift 'df' { id=5 }, marker 'mk' { id=6 },
}
print(seq[ 1].name) -- display: $start (start marker)
print(seq[-1].name) -- display: $end   (end   marker)

print(#seq.df, seq.df[3].id)                      -- display: 3   5
for _,e in ipairs(seq.df) do io.write(e.id," ") end -- display: 1 3 5
for _,e in ipairs(seq.mk) do io.write(e.id," ") end -- display: 2 4 6

-- print name of drift with id=3 in absolute and relative to id=6.
print(seq:name_of(4))       -- display: df[2]  (2nd df from start)
print(seq:name_of(2, -2))   -- display: df{-3} (3rd df before last mk)
```

The last two lines of code display the name of the same element but mangled with absolute and relative counts.

8 Iterators and ranges

Ranging a sequence triggers a complex look up mechanism where the arguments will be interpreted in various ways as described in the `:range_of` method, itself based on the methods `:index_of` and `:index`. The number of elements selected by a sequence range can be computed by the `:length_of` method, which accepts an extra *number* of turns to consider in the calculation.

The sequence iterators are created by the methods `:iter` and `:siter`, and both are based on the `:range_of` method as mentioned in their descriptions and includes an extra *number* of turns as for the method `:length_of`, and a direction `1` (forward) or `-1` (backward) for the iteration. The `:siter` differs from the `:iter` by its loop, which returns not only the sequence elements but also *implicit* drifts built on-the-fly when a gap $> 10^{-12}$ m is detected between two sequence elements. Such implicit drift have half-integer indexes and make the iterator "continuous" in s-positions.

The method `:foreach` uses the iterator returned by `:iter` with a range as its sole argument to loop over the elements where to apply the predicate before executing the action. The methods `:select`, `:deselect`, `:filter`, `:install`, `:replace`, `:remove`, `:move`, and `:misalign` are all based directly or indirectly on the `:foreach` method. Hence, to iterate backward over a sequence range, these methods have to use either its *list* form or a numerical range. For example the invocation `seq:foreach(\e -> print(e.name), {-2, 2, 'idx', -1})` will iterate backward over the entire sequence `seq` excluding the `$start` and `$end` markers, while the invocation `seq:foreach(\e -> print(e.name), 5..2..-1)` will iterate backward over the elements with s-positions sitting in the interval $[2, 5]$ m.

The tracking commands `survey` and `track` use the iterator returned by `:siter` for their main loop, with their `range`, `nturn` and `dir` attributes as arguments. These commands also save the iterator states in their `mflw` to allow the users to run them `nstep` by `nstep`, see commands `survey` and `track` for details.

The following example shows how to access to the elements with the `:foreach` method:

```
local sequence, drift, marker in MAD.element
local observed in MAD.element.flags
local seq = sequence {
  drift 'df' { id=1 }, marker 'mk' { id=2 },
  drift 'df' { id=3 }, marker 'mk' { id=4 },
  drift 'df' { id=5 }, marker 'mk' { id=6 },
}

local act = \e -> print(e.name,e.id)
seq:foreach(act, "df[2]/mk[3]")
-- display:
df   3
mk   4
df   5
mk   6

seq:foreach{action=act, range="df[2]/mk[3]", class=marker}
-- display: markers at ids 4 and 6
seq:foreach{action=act, pattern="^[^$]"}
-- display: all elements except $start and $end markers
seq:foreach{action=\e -> e:select(observed), pattern="mk"}
-- same as: seq:select(observed, {pattern="mk"})

local act = \e -> print(e.name, e.id, e:is_observed())
seq:foreach{action=act, range="#s/#e"}
-- display:
$start    nil   false
df        1     false
mk        2     true
df        3     false
mk        4     true
df        5     false
mk        6     true
$end      nil   true
```

9 Examples

9.1 FODO cell

The following example shows how to build a very simple FODO cell and an arc made of 10 FODO cells.

```
local sequence, sbend, quadrupole, sextupole, hkicker, vkicker, marker in MAD.element
local mkf = marker 'mkf' {}
local ang=2*math.pi/80
local fodo = sequence 'fodo' { refer='entry',
  mkf             { at=0, shared=true      }, -- mark the start of the fodo
  quadrupole 'qf' { at=0, l=1  , k1=0.3    },
  sextupole  'sf' {       l=0.3, k2=0      },
  hkicker    'hk' {       l=0.2, kick=0    },
  sbend      'mb' { at=2, l=2  , angle=ang },

  quadrupole 'qd' { at=5, l=1  , k1=-0.3   },
  sextupole  'sd' {       l=0.3, k2=0      },
  vkicker    'vk' {       l=0.2, kick=0    },
  sbend      'mb' { at=7, l=2  , angle=ang },
}
local arc = sequence 'arc' { refer='entry', 10*fodo }
fodo:dumpseq() ; print(fodo.mkf, mkf)
-- display:
sequence: fodo, l=9
idx  kind          name     l          dl       spos       upos    uds
001  marker        $start   0.000       0       0.000      0.000   0.000
002  marker        mkf      0.000       0       0.000      0.000   0.000
003  quadrupole    qf       1.000       0       0.000      0.000   0.000
004  sextupole     sf       0.300       0       1.000      1.000   0.000
005  hkicker       hk       0.200       0       1.300      1.300   0.000
006  sbend         mb       2.000       0       2.000      2.000   0.000
007  quadrupole    qd       1.000       0       5.000      5.000   0.000
008  sextupole     sd       0.300       0       6.000      6.000   0.000
009  vkicker       vk       0.200       0       6.300      6.300   0.000
010  sbend         mb       2.000       0       7.000      7.000   0.000
011  marker        $end     0.000       0       9.000      9.000   0.000
marker: 'mkf' 0x01015310e8  marker: 'mkf' 0x01015310e8 -- same marker
```

9.2 SPS compact description

The following dummy example shows a compact definition of the SPS mixing elements, beam lines and sequence definitions. The elements are zero-length, so the lattice is too.

```
local drift, sbend, quadrupole, bline, sequence in MAD.element

-- elements (empty!)
local ds = drift      'ds' {}
local dl = drift      'dl' {}
local dm = drift      'dm' {}
local b1 = sbend      'b1' {}
local b2 = sbend      'b2' {}
local qf = quadrupole 'qf' {}
local qd = quadrupole 'qd' {}

-- subsequences
local pf  = bline 'pf'  {qf,2*b1,2*b2,ds}           -- #: 6
local pd  = bline 'pd'  {qd,2*b2,2*b1,ds}           -- #: 6
local p24 = bline 'p24' {qf,dm,2*b2,ds,pd}          -- #: 11 (5+6)
local p42 = bline 'p42' {pf,qd,2*b2,dm,ds}          -- #: 11 (6+5)
local p00 = bline 'p00' {qf,dl,qd,dl}              -- #: 4
local p44 = bline 'p44' {pf,pd}                    -- #: 12 (6+6)
local insert = bline 'insert' {p24,2*p00,p42}      -- #: 30 (11+2*4+11)
local super  = bline 'super'  {7*p44,insert,7*p44}  -- #: 198 (7*12+30+7*12)

-- final sequence
local SPS = sequence 'SPS' {6*super}               -- # = 1188 (6*198)

-- check number of elements and length
print(#SPS, SPS.l)  -- display: 1190  0 (no element length provided)
```

9.3 Installing elements I

The following example shows how to install elements and subsequences in an empty initial sequence:

```
local sequence, drift in MAD.element
local seq   = sequence "seq" { l=16, refer="entry", owner=true }
local sseq1 = sequence "sseq1" {
  at=5, l=6 , refpos="centre", refer="entry",
  drift "df1'" {l=1, at=-4, from="end"},
  drift "df2'" {l=1, at=-2, from="end"},
  drift "df3'" {     at= 5            },
}
local sseq2 = sequence "sseq2" {
  at=14, l=6, refpos="exit", refer="entry",
  drift "df1''" { l=1, at=-4, from="end"},
  drift "df2''" { l=1, at=-2, from="end"},
  drift "df3''" {      at= 5            },
}
seq:install {
  drift "df1" {l=1, at=1},
  sseq1, sseq2,
  drift "df2" {l=1, at=15},
} :dumpseq()

-- display:
sequence: seq, l=16
idx  kind          name       l          dl       spos       upos    uds
001  marker        $start     0.000       0       0.000       0.000   0.000
002  drift         df1        1.000       0       1.000       1.000   0.000
003  drift         df1'       1.000       0       4.000       4.000   0.000
004  drift         df2'       1.000       0       6.000       6.000   0.000
005  drift         df3'       0.000       0       7.000       7.000   0.000
006  drift         df1''      1.000       0      10.000      10.000   0.000
007  drift         df2''      1.000       0      12.000      12.000   0.000
008  drift         df3''      0.000       0      13.000      13.000   0.000
009  drift         df2        1.000       0      15.000      15.000   0.000
010  marker        $end       0.000       0      16.000      16.000   0.000
```

9.4 Installing elements II

The following more complex example shows how to install elements and subsequences in a sequence using a selection and the packed form for arguments:

```
local mk   = marker   "mk"  { }
local seq  = sequence "seq" { l = 10, refer="entry",
  mk "mk1" { at = 2 },
  mk "mk2" { at = 4 },
  mk "mk3" { at = 8 },
}
local sseq = sequence "sseq" { l = 3 , at = 5, refer="entry",
  drift "df1'" { l = 1, at = 0 },
  drift "df2'" { l = 1, at = 1 },
  drift "df3'" { l = 1, at = 2 },
}
seq:install {
  class    = mk,
  elements = {
    drift "df1" { l = 0.1, at = 0.1, from="selected" },
    drift "df2" { l = 0.1, at = 0.2, from="selected" },
    drift "df3" { l = 0.1, at = 0.3, from="selected" },
    sseq,
    drift "df4" { l = 1, at = 9 },
  }
}

seq:dumpseq()
-- display:
sequence: seq, l=10
idx  kind          name      l          dl       spos       upos    uds
001  marker        $start    0.000       0       0.000      0.000   0.000
002  marker        mk1       0.000       0       2.000      2.000   0.000
003  drift         df1       0.100       0       2.100      2.100   0.000
004  drift         df2       0.100       0       2.200      2.200   0.000
005  drift         df3       0.100       0       2.300      2.300   0.000
006  marker        mk2       0.000       0       4.000      4.000   0.000
007  drift         df1       0.100       0       4.100      4.100   0.000
008  drift         df2       0.100       0       4.200      4.200   0.000
009  drift         df3       0.100       0       4.300      4.300   0.000
010  drift         df1'      1.000       0       5.000      5.000   0.000
011  drift         df2'      1.000       0       6.000      6.000   0.000
012  drift         df3'      1.000       0       7.000      7.000   0.000
013  marker        mk3       0.000       0       8.000      8.000   0.000
014  drift         df1       0.100       0       8.100      8.100   0.000
015  drift         df2       0.100       0       8.200      8.200   0.000
016  drift         df3       0.100       0       8.300      8.300   0.000
017  drift         df4       1.000       0       9.000      9.000   0.000
018  marker        $end      0.000       0      10.000     10.000   0.000
```

Chapter 8. MTables

The MAD Tables (MTables) — also named Table File System (TFS) — are objects convenient to store, read and write a large amount of heterogeneous information organized as columns and header. The MTables are also containers that provide fast access to their rows, columns, and cells by referring to their indexes, or some values of the designated reference column, or by running iterators constrained with ranges and predicates.

The `mtable` object is the *root object* of the TFS tables that store information relative to tables.

The `mtable` module extends the `typeid` module with the `is_mtable` function, which returns `true` if its argument is a `mtable` object, `false` otherwise.

1 Attributes

The `mtable` object provides the following attributes:

`type`	A *string* specifying the type of the mtable (often) set to the name of the command that created it, like `survey`, `track` or `twiss`. (default: `'user'`).
`title`	A *string* specifying the title of the mtable (often) set to the attribute `title` of the command that created it. (default: `'no-title'`).
`origin`	A *string* specifying the origin of the mtable. (default: `"MAD` *`version os arch`*`"`).
`date`	A *string* specifying the date of creation of the mtable. (default: `"`*`day/month/year`*`"`).
`time`	A *string* specifying the time of creation of the mtable. (default: `"`*`hour`*`:`*`min`*`:`*`sec`*`"`).
`refcol`	A *string* specifying the name of the reference column used to build the dictionary of the mtable, and to mangle values with counts. (default: `nil`).
`header`	A *list* specifying the augmented attributes names (and their order) used by default for the header when writing the mtable to files. Augmented meaning that the *list* is concatenated to the *list* held by the parent mtable during initialization. (default: `{'name', 'type', 'title', 'origin', 'date', 'time', 'refcol'}`).
`column`	A *list* specifying the augmented columns names (and their order) used by default for the columns when writing the mtable to files. Augmented meaning that the *list* is concatenated to the *list* held by the parent mtable during initialization. (default: `nil`).
`novector`	A *logical* specifying to not convert (`novector == true`) columns containing only numbers to vectors during the insertion of the second row. The attribute can also be a *list* specifying the columns names to remove from the specialization. If the *list* is empty or `novector ~= true`, all numeric columns will be converted to vectors, and support all methods and operations from the linear algebra module. (default: `nil`).
`owner`	A *logical* specifying if an *empty* mtable is a view with no data (`owner ~= true`), or a mtable holding data (`owner == true`). (default: `nil`).
`reserve`	A *number* specifying an estimate of the maximum number of rows stored in the mtable. If the value is underestimated, the mtable will still expand on need. (default: `8`).

Warning: the following private and read-only attributes are present in all mtables and should *never be used, set or changed*; breaking this rule would lead to an *undefined behavior*:

`__dat` A *table* containing all the private data of mtables.

`__seq` A *sequence* attached to the mtable by the `survey` and `track` commands and used by the methods receiving a *reference* to an element as argument. (default: `nil`).

`__cycle` A *reference* to the row registered with the `:cycle` method. (default: `nil`).

2 Methods

The `mtable` object provides the following methods:

`nrow` A *method* `()` returning the *number* of rows in the mtable.

`ncol` A *method* `()` returning the *number* of columns in the mtable.

`ngen` A *method* `()` returning the *number* of columns generators in the mtable. The *number* of columns with data is given by `:ncol() - :ngen()`.

`colname` A *method* `(idx)` returning the *string* name of the `idx`-th column in the mtable or `nil`.

`colnames` A *method* `([lst])` returning the *list* `lst` (default: `{}`) filled with all the columns names of the mtable.

`index` A *method* `(idx)` returning a positive index, or `nil`. If `idx` is negative, it is reflected versus the size of the mtable, e.g. `-1` becomes `#self`, the index of the last row.

`name_of` A *method* `(idx, [ref])` returning a *string* corresponding to the (mangled) *value* from the reference column of the row at the index `idx`, or `nil`. A row *value* appearing more than once in the reference column will be mangled with an absolute count, e.g. `mq[3]`, or a relative count versus the reference row determined by `:index_of(ref)`, e.g. `mq{-2}`.

`index_of` A *method* `(a, [ref], [dir])` returning a *number* corresponding to the positive index of the row determined by the first argument or `nil`. If `a` is a *number* (or a *string* representing a *number*), it is interpreted as the index of the row and returned as a second *number*. If `a` is a *string*, it is interpreted as the (mangled) *value* of the row in the reference column as returned by `:name_of`. Finally, `a` can be a *reference* to an element to search for **if** the mtable has both, an attached sequence, and a column named `'eidx'` mapping the indexes of the elements to the attached sequence.[1] The argument `ref` (default: `nil`) specifies the reference row determined by `:index_of(ref)` to use for relative indexes, for decoding mangled values with relative counts, or as the reference row to start searching from. The argument `dir` (default: `1`) specifies the direction of the search with values `1` (forward), `-1` (backward), or `0` (no direction), which correspond respectively to the rounding methods `ceil`, `floor` and `round` from the `gmath` module.

`range_of` A *method* `([rng], [ref], [dir])` returning three *numbers* corresponding to the positive indexes *start* and *end* of the range and its direction *dir* (default: `1`), or `nil` for an empty range. If `rng` is omitted, it returns `1`, `#self`, `1`, or `#self`, `1`, `-1` if `dir` is negative. If `rng` is a *number* or a *string* with no `'/'` separator, it is interpreted as *start* and *end*, both determined by `:index_of`. If `rng` is a *string* containing the separator `'/'`, it is split in two *strings* interpreted as *start* and *end*, both determined by `:index_of`. If `rng` is a *list*, it will be interpreted as { *start*, *end*, `[ref]`, `[dir]` }, both determined by `:index_of`. The arguments `ref` and `dir` are forwarded to all invocations of `:index_of`

[1] These information are usually provided by the command creating the `mtable`, like `survey` and `track`.

with a higher precedence than ones in the *list* `rng`, and a runtime error is raised if the method returns `nil`, i.e. to disambiguate between a valid empty range and an invalid range.

`length_of` A *method* `([rng], [ntrn], [dir])` returning a *number* specifying the length of the range optionally including `ntrn` extra turns (default: `0`), and calculated from the indexes returned by `:range_of([rng], nil, [dir])`.

`get` A *method* `(row, col, [cnt])` returning the *value* stored in the mtable at the cell `(row,col)`, or `nil`. If `row` is a not a row index determined by `:index(row)`, it is interpreted as a (mangled) *value* to search in the reference column, taking into account the count `cnt` (default: `1`). If `col` is not a column index, it is interpreted as a column name.

`set` A *method* `(row, col, val, [cnt])` returning the mtable itself after updating the cell `(row,col)` to the value `val`, or raising an error if the cell does not exist. If `row` is a not a row index determined by `:index(row)`, it is interpreted as a (mangled) *value* to search in the reference column, taking into account the count `cnt` (default: `1`). If `col` is not a column index, it is interpreted as a column name.

`getcol` A *method* `(col)` returning the column `col`, or `nil`. If `col` is not a column index, it is interpreted as a column name.

`setcol` A *method* `(col, val)` returning the mtable itself after updating the column `col` with the values of `val`, or raising an error if the column does not exist. If `col` is not a column index, it is interpreted as a column name. If the column is a generator, so must be `val` or an error will be raised. If the column is not a generator and `val` is a *callable* `(ri)`, it will be invoked with the row index `ri` as its sole argument, using its returned value to update the column cell. Otherwise `val` must be an *iterable* or an error will be raised. If the column is already a specialized *vector*, the *iterable* must provide enough numbers to fill it entirely as `nil` is not a valid value.

`inscol` A *method* `([ref], col, val, [nvec])` returning the mtable itself after inserting the column data `val` with the *string* name `col` at index `ref` (default: `:ncol()+1`). If `ref` is not a column index, it is interpreted as a column name. If `val` is a *callable* `(ri)`, it will be added as a column generator. Otherwise `val` must be an *iterable* or an error will be raised. The *iterable* will used to fill the new column that will be specialized to a *vector* if its first value is a *number* and `nvec ~= true` (default: `nil`).

`addcol` A *method* `(col, val, [nvec])` equivalent to `:inscol(nil, col, val, [nvec])`.

`remcol` A *method* `(col)` returning the mtable itself after removing the column `col`, or raising an error if the column does not exist. If `col` is not a column index, it is interpreted as a column name.

`rencol` A *method* `(col, new)` returning the mtable itself after renaming the column `col` to the *string* `new`, or raising an error if the column does not exist. If `col` is not a column index, it is interpreted as a column name.

`getrow` A *method* `(row, [ref])` returning the *mappable* (proxy) of the row determined by the method `:index_of(row, [ref])`, or `nil`.

`setrow` A *method* `(row, val, [ref])` returning the mtable itself after updating the row at index determined by `:index_of(row, [ref])` using the values provided by the *mappable* `val`, which can be a *list* iterated as pairs of (*index*, *value*) or a *set* iterated as pairs of (*key*, *value*) with *key* being the column names, or a combination of the two. An error is raised if the column does not exist.

`insrow` A *method* `(row, val, [ref])` returning the mtable itself after inserting a new row at index determined by `:index_of(row, [ref])` and filled with the values provided by the *mappable* `val`, which can be a *list* iterated as pairs of (*index*, *value*) or a *set* iterated as pairs of (*key*, *value*) with *key* being the column names or a combination of the two.

`addrow` A *method* `(val)` equivalent to `:insrow(#self+1, val)`.

`remrow` A *method* `(row, [ref])` returning the mtable itself after removing the row determined by the method `:index_of(row, [ref])`, or raising an error if the row does not exist.

`swprow` A *method* `(row1, row2, [ref1], [ref2])` returning the mtable itself after swapping the content of the rows, both determined by the method `:index_of(row, [ref])`, or raising an error if one of the row does not exist.

`clrrow` A *method* `(row, [ref])` returning the mtable itself after clearing the row determined by the method `:index_of(row, [ref])`, or raising an error if the row does not exist; where clearing the row means to set *vector* value to `0` and `nil` otherwise.

`clear` A *method* `()` returning the mtable itself after clearing all the rows, i.e. `#self == 0`, with an opportunity for new columns specialization.

`iter` A *method* `([rng], [ntrn], [dir])` returning an iterator over the mtable rows. The optional range is determined by `:range_of([rng], [dir])`, optionally including `ntrn` turns (default: `0`). The optional direction `dir` specifies the forward `1` or the backward `-1` direction of the iterator. If `rng` is not provided and the mtable is cycled, the *start* and *end* indexes are determined by `:index_of(self.__cycle)`. When used with a generic **`for`** loop, the iterator returns at each rows the index and the row *mappable* (proxy).

`foreach` A *method* `(act, [rng], [sel], [not])` returning the mtable itself after applying the action `act` on the selected rows. If `act` is a *set* representing the arguments in the packed form, the missing arguments will be extracted from the attributes `action`, `range`, `select` and `default`. The action `act` must be a *callable* `(row, idx)` applied to a row passed as first argument and its index as second argument. The optional range is used to generate the loop iterator `:iter([rng])`. The optional selector `sel` is a *callable* `(row, idx)` predicate selecting eligible rows for the action from the row itself passed as first argument and its index as second argument. The selector `sel` can be specified in other ways, see row selections for details. The optional *logical* `not` (default: `false`) indicates how to interpret default selection, as *all* or *none*, depending on the semantic of the action.[2]

`select` A *method* `([rng], [sel], [not])` returning the mtable itself after selecting the rows using `:foreach(sel_act, [rng], [sel], [not])`. By default mtable have all their rows deselected, the selection being stored as *boolean* in the column at index `0` and named `is_selected`.

`deselect` A *method* `([rng], [sel], [not])` returning the mtable itself after deselecting the rows using `:foreach(desel_act, [rng], [sel], [not])`. By default mtable have all their rows deselected, the selection being stored as *boolean* in the column at index `0` and named `is_selected`.

[2]For example, the `:remove` method needs `not=true` to *not* remove all rows if no selector is provided.

filter A *method* `([rng], [sel], [not])` returning a *list* containing the positive indexes of the rows determined by `:foreach(filt_act, [rng], [sel], [not])`, and its size.

insert A *method* `(row, [rng], [sel])` returning the mtable itself after inserting the rows in the *list* `row` at the indexes determined by `:filter([rng], [sel], true)`. If the arguments are passed in the packed form, the extra attribute `rows` will be used as a replacement for the argument `row`, and if the attribute `where="after"` is defined then the rows will be inserted after the selected indexes. The insertion scheme depends on the number R of rows in the *list* `row` versus the number S of rows selected by `:filter`; 1×1 (one row inserted at one index), $R \times 1$ (R rows inserted at one index), $1 \times S$ (one row inserted at S indexes) and $R \times S$ (R rows inserted at S indexes). Hence, the insertion schemes insert respectively 1, R, S, and $\min(R, S)$ rows.

remove A *method* `([rng], [sel])` returning the mtable itself after removing the rows determined by `:filter([rng], [sel], true)`.

sort A *method* `(cmp, [rng], [sel])` returning the mtable itself after sorting the rows at the indexes determined by `:filter([rng], [sel], true)` using the ordering *callable* `cmp(row1, row2)`. The arguments `row1` and `row2` are *mappable* (proxies) referring to the current rows being compared and providing access to the columns values for the comparison.[3] The argument `cmp` can be specified in a compact ordering form as a *string* that will be converted to an ordering *callable* by the function `str2cmp` from the utility module. For example, the *string* "-y,x" will be converted by the method to the following *lambda* `\r1,r2 -> r1.y > r2.y or r1.y == r2.y and r1.x < r2.x`, where `y` and `x` are the columns used to sort the mtable in descending (−) and ascending (+) order respectively. The compact ordering form is not limited in the number of columns and avoids making mistakes in the comparison logic when many columns are involved.

cycle A *method* `(a)` returning the mtable itself after checking that `a` is a valid reference using `:index_of(a)`, and storing it in the `__cycle` attribute, itself erased by the methods editing the mtable like `:insert`, `:remove` or `:sort`.

copy A *method* `([name], [owner])` returning a new mtable from a copy of `self`, with the optional `name` and the optional attribute `owner` set. If the mtable is a view, so will be the copy unless `owner == true`.

is_view A *method* `()` returning `true` if the mtable is a view over another mtable data, `false` otherwise.

set_readonly Set the mtable as read-only, including the columns and the rows proxies.

read A *method* `([filname])` returning a new instance of `self` filled with the data read from the file determined by `openfile(filename, 'r', {'.tfs','.txt','.dat'})` from the utility module. This method can read columns containing the data types *nil*, *boolean*, *number*, *complex number*, (numerical) *range*, and (quoted) *string*. The header can also contain tables saved as *string* and decoded with *function* `str2tbl` from the utility module.

write A *method* `([filname], [clst], [hlst], [rsel])` returning the mtable itself after writing its content to the file determined by `openfile(filename, 'w', {'.tfs', '.txt', '.dat'})` from the utility module. The columns to write and their order is determined by `clst` or `self.column` (default: `nil` ≡ all columns). The attributes to write

[3] A *mappable* supports the length operator `#`, the indexing operator `[]`, and generic `for` loops with `pairs`.

in the header and their order is determined by `hlst` or `self.header`. The *logical* `rsel` indicates to save all rows or only rows selected by the `:select` method (`rsel == true`). This method can write columns containing the data types *nil*, *boolean*, *number*, *complex number*, (numerical) *range*, and (quoted) *string*. The header can also contain tables saved as *string* and encoded with *function* `tbl2str` from the utility module.

`print` A *method* `([clst], [hlst], [rsel])` equivalent to `:write(nil, [clst], [hlst], [rsel])`.

`save_sel` A *method* `([sel])` saving the rows selection to the optional *iterable* `sel` (default: `{}`) and return it.

`restore_sel` A *method* `(sel)` restoring the rows selection from the *iterable* `sel`. The indexes of `sel` must match the indexes of the rows in the mtable.

`make_dict` A *method* `([col])` returning the mtable itself after building the rows dictionnary from the values of the reference column determined by `col` (default: `refcol`) for fast row access. If `col` is not a column index, it is interpreted as a column name except for the special name `'none'` that disables the rows dictionnary and reset `refcol` to `nil`.

`check_mtbl` A *method* `()` checking the integrity of the mtable and its dictionary (if any), for debugging purpose only.

3 Metamethods

The `mtable` object provides the following metamethods:

`__len` A *metamethod* `()` called by the length operator `#` to return the number of rows in the mtable.

`__add` A *metamethod* `(val)` called by the plus operator `+` returning the mtable itself after appending the row `val` at its end, similiar to the `:addrow` method.

`__index` A *metamethod* `(key)` called by the indexing operator `[key]` to return the *value* of an attribute determined by *key*. The *key* is interpreted differently depending on its type with the following precedence:

1. A *number* is interpreted as a row index and returns an *iterable* on the row (proxy) or `nil`.
2. Other *key* types are interpreted as *object* attributes subject to object model lookup.
3. If the *value* associated with *key* is `nil`, then *key* is interpreted as a column name and returns the column if it exists, otherwise...
4. If *key* is not a column name, then *key* is interpreted as a value in the reference column and returns either an *iterable* on the row (proxy) determined by this value or an *iterable* on the rows (proxies) holding this non-unique value.[4]
5. Otherwise returns `nil`.

`__newindex` A *metamethod* `(key, val)` called by the assignment operator `[key]=val` to create new attributes for the pairs (*key*, *value*). If *key* is a *number* or a value specifying a row in the reference column or a *string* specifying a column name, the following error is raised:

[4]An *iterable* supports the length operator `#`, the indexing operator `[]`, and generic `for` loops with `ipairs`.

```
"invalid mtable write access (use 'set' methods)"
```

__init A *metamethod* `()` called by the constructor to build the mtable from the column names stored in its *list* part and some attributes, like `owner`, `reserve` and `novector`.

__copy A *metamethod* `()` similar to the *method* `copy`.

The following attribute is stored with metamethods in the metatable, but has different purpose:

__mtbl A unique private *reference* that characterizes mtables.

4 MTables creation

During its creation as an *object*, a mtable can defined its attributes as any object, and the *list* of its column names, which will be cleared after its initialization. Any column name in the *list* that is enclosed by braces is designated as the refererence column for the dictionnary that provides fast row indexing, and the attribute `refcol` is set accordingly.

Some attributes are considered during the creation by the *metamethod* `__init`, like `owner`, `reserve` and `novector`, and some others are initialized with defined values like `type`, `title`, `origin`, `date`, `time`, and `refcol`. The attributes `header` and `column` are concatenated with the the parent ones to build incrementing *list* of attributes names and columns names used by default when writing the mtable to files, and these lists are not provided as arguments.

The following example shows how to create a mtable form a *list* of column names add rows:

```
local mtable in MAD
local tbl = mtable 'mytable' {

   {'name'}, 'x', 'y' } -- column 'name' is the refcol
  + { 'p11', 1.1, 1.2 }
  + { 'p12', 2.1, 2.2 }
  + { 'p13', 2.1, 3.2 }
  + { 'p11', 3.1, 4.2 }
print(tbl.name, tbl.refcol, tbl:getcol'name')
-- display: mytable  name   mtable reference column: 0x010154cd10
```

Pitfall: When a column is named `'name'`, it must be explicitly accessed, e.g. with the `:getcol` method, as the indexing operator `[]` gives the precedence to object's attributes and methods. Hence, `tbl.name` returns the table name `'mytable'`, not the column `'name'`.

5 Rows selections

The row selection in mtable use predicates in combination with iterators. The mtable iterator manages the range of rows where to apply the selection, while the predicate says if a row in this range is illegible for the selection. In order to ease the use of methods based on the `:foreach` method, the selector predicate `sel` can be built from different types of information provided in a *set* with the following attributes:

selected A *boolean* compared to the rows selection stored in column `'is_selected'`.

pattern A *string* interpreted as a pattern to match the *string* in the reference column, which must exist, using `string.match` from the standard library, see Lua 5.2 §6.4 for details. If the reference column does not exist, it can be built using the `:make_dict` method.

`list` An *iterable* interpreted as a *list* used to build a *set* and select the rows by their name, i.e. the built predicate will use `tbl[row.name]` as a *logical*, meaning that column `name` must exists. An alternate column name can be provided through the key `colname`, i.e. used as `tbl[row[colname]]`. If the *iterable* is a single item, e.g. a *string*, it will be converted first to a *list*.

`table` A *mappable* interpreted as a *set* used to select the rows by their name, i.e. the built predicate will use `tbl[row.name]` as a *logical*, meaning that column `name` must exists. If the *mappable* contains a *list* or is a single item, it will be converted first to a *list* and its *set* part will be discarded.

`kind` An *iterable* interpreted as a *list* used to build a *set* and select the rows by their kind, i.e. the built predicate will use `tbl[row.kind]` as a *logical*, meaning that column `kind` must exists. If the *iterable* is a single item, e.g. a *string*, it will be converted first to a *list*. This case is equivalent to `list` with `colname='kind'`.

`select` A *callable* interpreted as the selector itself, which allows to build any kind of predicate or to complete the restrictions already built above.

All these attributes are used in the aforementioned order to incrementally build predicates that are combined with logical conjunctions, i.e. `and`'ed, to give the final predicate used by the `:foreach` method. If only one of these attributes is needed, it is possible to pass it directly in `sel`, not as an attribute in a *set*, and its type will be used to determine the kind of predicate to build. For example, `tbl:foreach(act, "^MB")` is equivalent to `tbl:foreach{action=act, pattern="^MB"}`.

6 Indexes, names and counts

Indexing a mtable triggers a complex look up mechanism where the arguments will be interpreted in various ways as described in the metamethod `__index`. A *number* will be interpreted as a relative row index in the list of rows, and a negative index will be considered as relative to the end of the mtable, i.e. `-1` is the last row. Non-*number* will be interpreted first as an object key (can be anything), looking for mtable methods or attributes; then as a column name or as a row *value* in the reference column if nothing was found.

If a row exists but its *value* is not unique in the reference column, an *iterable* is returned. An *iterable* supports the length `#` operator to retrieve the number of rows with the same *value*, the indexing operator `[]` waiting for a count n to retrieve the n-th row from the start with that *value*, and the iterator `ipairs` to use with generic `for` loops.

The returned *iterable* is in practice a proxy, i.e. a fake intermediate object that emulates the expected behavior, and any attempt to access the proxy in another manner should raise a runtime error.

Note: Compared to the sequence, the indexing operator `[]` and the method `:index_of` of the mtable always interprets a *number* as a (relative) row index. To find a row from a s-position [m] in the mtable if the column exists, use the functions `lsearch` or `bsearch` (if they are monotonic) from the utility module.

The following example shows how to access to the rows through indexing and the *iterable*:

```
local mtable in MAD
local tbl = mtable { {'name'}, 'x', 'y' } -- column 'name' is the refcol
                   + { 'p11', 1.1, 1.2 }
                   + { 'p12', 2.1, 2.2 }
                   + { 'p13', 2.1, 3.2 }
                   + { 'p11', 3.1, 4.2 }
print(tbl[ 1].y) -- display: 1.2
print(tbl[-1].y) -- display: 4.2

print(#tbl.p11, tbl.p12.y, tbl.p11[2].y)           -- display: 2 2.2 4.2
for _,r in ipairs(tbl.p11) do io.write(r.x," ") end -- display: 1.1 3.1
for _,v in ipairs(tbl.p12) do io.write(v,  " ") end -- display: 'p12' 2.1 2.2

-- print name of point with name p11 in absolute and relative to p13.
print(tbl:name_of(4))       -- display: p11[2]  (2nd p11 from start)
print(tbl:name_of(1, -2))   -- display: p11{-1} (1st p11 before p13)
```

The last two lines of code display the name of the same row but mangled with absolute and relative counts.

7 Iterators and ranges

Ranging a mtable triggers a complex look up mechanism where the arguments will be interpreted in various ways as described in the method `:range_of`, itself based on the methods `:index_of` and `:index`. The number of rows selected by a mtable range can be computed by the `:length_of` method, which accepts an extra *number* of turns to consider in the calculation.

The mtable iterators are created by the method `:iter`, based on the method `:range_of` as mentioned in its description and includes an extra *number* of turns as for the method `:length_of`, and a direction `1` (forward) or `-1` (backward) for the iteration.

The method `:foreach` uses the iterator returned by `:iter` with a range as its sole argument to loop over the rows where to apply the predicate before executing the action. The methods `:select`, `:deselect`, `:filter`, `:insert`, and `:remove` are all based directly or indirectly on the `:foreach` method. Hence, to iterate backward over a mtable range, these methods have to use either its *list* form or a numerical range. For example the invocation `tbl:foreach(\r -> print(r.name), {-2, 2, nil, -1})` will iterate backward over the entire mtable excluding the first and last rows, equivalently to the invocation `tbl:foreach(\r -> print(r.name), -2..2..-1)`.

The following example shows how to access to the rows with the `:foreach` method:

```
local mtable in MAD
local tbl = mtable { {'name'}, 'x', 'y' }
                   + { 'p11', 1.1, 1.2 }
                   + { 'p12', 2.1, 2.2 }
                   + { 'p13', 2.1, 3.2 }
                   + { 'p11', 3.1, 4.2 }

local act = \r -> print(r.name, r.y)
tbl:foreach(act, -2..2..-1)
-- display:  p13   3.2
!            p12   2.2
tbl:foreach(act, "p11[1]/p11[2]")
-- display:  p11   1.2
!            p12   2.2
!            p13   3.2
!            p11   4.2
tbl:foreach{action=act, range="p11[1]/p13"}
-- display:  p11   1.2
!            p12   2.2
!           p13   3.2
tbl:foreach{action=act, pattern="[^1]$"}
-- display:  p12   2.2
!           p13   3.2
local act = \r -> print(r.name, r.y, r.is_selected)
tbl:select{pattern="p.1"}:foreach{action=act, range="1/-1"}
-- display:  p11   1.2   true
!           p12   2.2   nil
!           p13   3.2   nil
!           p11   4.2   true
```

8 Examples

8.1 Creating a MTable

The following example shows how the `track` command, i.e. `self` hereafter, creates its MTable:

```
local header = { -- extra attributes to save in track headers
  'direction', 'observe', 'implicit', 'misalign', 'deltap', 'lost' }

local function make_mtable (self, range, nosave)
  local title, dir, observe, implicit, misalign, deltap, savemap in self
  local sequ, nrow = self.sequence, nosave and 0 or 16

  return mtable(sequ.name, { -- keep column order!
    type='track', title=title, header=header,
    direction=dir, observe=observe, implicit=implicit, misalign=misalign,
    deltap=deltap, lost=0, range=range, reserve=nrow, __seq=sequ,
    {'name'}, 'kind', 's', 'l', 'id', 'x', 'px', 'y', 'py', 't', 'pt',
    'slc', 'turn', 'tdir', 'eidx', 'status', savemap and '__map' or nil })
end
```

8.2 Extending a MTable

The following example shows how to extend the MTable created by a `twiss` command with the elements tilt, angle and integrated strengths from the attached sequence:

```
-- The prelude creating the sequence seq is omitted.
local tws = twiss { sequence=seq, method=4, cofind=true }

local is_integer in MAD.typeid
tws:addcol('angle', \ri => -- add angle column
      local idx = tws[ri].eidx
      return is_integer(idx) and tws.__seq[idx].angle or 0 end)
   :addcol('tilt', \ri => -- add tilt column
      local idx = tws[ri].eidx
      return is_integer(idx) and tws.__seq[idx].tilt or 0 end)

for i=1,6 do -- add kil and kisl columns
tws:addcol('k'..i-1..'l', \ri =>
      local idx = tws[ri].eidx
      if not is_integer(idx) then return 0 end -- implicit drift
```

```
        local elm = tws.__seq[idx]
        return (elm['k'..i-1] or 0)*elm.l + ((elm.knl or {})[i] or 0)
      end)
     :addcol('k'..i-1..'sl', \ri =>
        local idx = tws[ri].eidx
        if not is_integer(idx) then return 0 end -- implicit drift
        local elm = tws.__seq[idx]
        return (elm['k'..i-1..'s'] or 0)*elm.l + ((elm.ksl or {})[i] or 0)
      end)
end

local cols = {'name', 'kind', 's', 'l', 'angle', 'tilt',
    'x', 'px', 'y', 'py', 't', 'pt',
    'beta11', 'beta22', 'alfa11', 'alfa22', 'mu1', 'mu2', 'dx', 'ddx',
    'k1l', 'k2l', 'k3l', 'k4l', 'k1sl', 'k2sl', 'k3sl', 'k4sl'}

tws:write("twiss", cols) -- write header and columns to file twiss.tfs
```

Hopefully, the physics module provides the *function* `melmcol(mtbl, cols)` to achieve the same task easily:

```
-- The prelude creating the sequence seq is omitted.
local tws = twiss { sequence=seq, method=4, cofind=true }

-- Add element properties as columns
local melmcol in MAD.gphys
local melmcol(tws, {'angle', 'tilt', 'k1l' , 'k2l' , 'k3l' , 'k4l',
                                         'k1sl', 'k2sl', 'k3sl', 'k4sl'})

-- write TFS table
tws:write("twiss", {
    'name', 'kind', 's', 'l', 'angle', 'tilt',
    'x', 'px', 'y', 'py', 't', 'pt',
    'beta11', 'beta22', 'alfa11', 'alfa22', 'mu1', 'mu2', 'dx', 'ddx',
    'k1l', 'k2l', 'k3l', 'k4l', 'k1sl', 'k2sl', 'k3sl', 'k4sl'})
```

Chapter 9. MADX

Part II

Commands

Chapter 10. Introduction

Chapter 11. Survey

The `survey` command provides a simple interface to the *geometric* tracking code.[1] The geometric tracking can be used to place the elements of a sequence in the global reference system.

1 Command synopsis

Figure 11.1: Synopsis of the `survey` command with default setup.

```
mtbl, mflw [, eidx] = survey {
  sequence=sequ,   -- sequence (required)
  range=nil,       -- range of tracking (or sequence.range)
  dir=1,           -- s-direction of tracking (1 or -1)
  s0=0,            -- initial s-position offset [m]
  X0=0,            -- initial coordinates x, y, z [m]
  A0=0,            -- initial angles theta, phi, psi [rad] or matrix W0
  nturn=1,         -- number of turns to track
  nstep=-1,        -- number of elements to track
  nslice=1,        -- number of slices (or weights) for each element
  implicit=false,  -- slice implicit elements too (e.g. plots)
  misalign=false,  -- consider misalignment
  save=true,       -- create mtable and save results
  title=nil,       -- title of mtable (default seq.name)
  observe=0,       -- save only in observed elements (every n turns)
  savesel=fnil,    -- save selector (predicate)
  savemap=false,   -- save the orientation matrix W in the column __map
  atentry=fnil,    -- action called when entering an element
  atslice=fnil,    -- action called after each element slices
  atexit=fnil,     -- action called when exiting an element
  atsave=fnil,     -- action called when saving in mtable
  atdebug=fnil,    -- action called when debugging the element maps
  info=nil,        -- information level (output on terminal)
  debug=nil,       -- debug information level (output on terminal)
  usrdef=nil,      -- user defined data attached to the mflow
  mflow=nil,       -- mflow, exclusive with other attributes except nstep
}
```

The `survey` command format is summarized in Figure 11.1, including the default setup of the attributes. The `survey` command supports the following attributes:

`sequence` The *sequence* to survey. (no default, required).
Example: `sequence = lhcb1`.

`range` A *range* specifying the span of the sequence survey. If no range is provided, the command looks for a range attached to the sequence, i.e. the attribute `seq.range`. (default: `nil`).
Example: `range = "S.DS.L8.B1/E.DS.R8.B1"`.

[1] MAD-NG implements only two tracking codes denominated the *geometric* and the *dynamic* tracking.

`dir`	The s-direction of the tracking: `1` forward, `-1` backward. (default: `1`). Example: `dir = -1`.
`s0`	A *number* specifying the initial s-position offset. (default: `0` [m]). Example: `s0 = 5000`.
`X0`	A *mappable* specifying the initial coordinates `{x,y,z}`. (default: `0` [m]). Example: `X0 = { x=100, y=-50 }`
`A0`	A *mappable* specifying the initial angles `theta`, `phi` and `psi` or an orientation *matrix* `W0`.[2] (default: `0` [rad]). Example: `A0 = { theta=deg2rad(30) }`
`nturn`	A *number* specifying the number of turn to track. (default: `1`). Example: `nturn = 2`.
`nstep`	A *number* specifying the number of element to track. A negative value will track all elements. (default: `-1`). Example: `nstep = 1`.
`nslice`	A *number* specifying the number of slices or an *iterable* of increasing relative positions or a *callable* `(elm, mflw, lw)` returning one of the two previous kind of positions to track in the elements. The arguments of the callable are in order, the current element, the tracked map flow, and the length weight of the step. This attribute can be locally overridden by the element. (default: `1`). Example: `nslice = 5`.
`implicit`	A *logical* indicating that implicit elements must be sliced too, e.g. for smooth plotting. (default: `false`). Example: `implicit = true`.
`misalign`	A *logical* indicating that misalignment must be considered. (default: `false`). Example: `implicit = true`.
`save`	A *logical* specifying to create a *mtable* and record tracking information at the observation points. The `save` attribute can also be a *string* specifying saving positions in the observed elements: `"atentry"`, `"atslice"`, `"atexit"` (i.e. `true`), `"atbound"` (i.e. entry and exit), `"atbody"` (i.e. slices and exit) and `"atall"`. (default:`true`). Example: `save = false`.
`title`	A *string* specifying the title of the *mtable*. If no title is provided, the command looks for the name of the sequence, i.e. the attribute `seq.name`. (default: `nil`). Example: `title = "Survey around IP5"`.
`observe`	A *number* specifying the observation points to consider for recording the tracking information. A zero value will consider all elements, while a positive value will consider selected elements only, checked with method `:is_observed`, every `observe`> 0 turns. (default: `0`). Example: `observe = 1`.
`savesel`	A *callable* `(elm, mflw, lw, islc)` acting as a predicate on selected elements for observation, i.e. the element is discarded if the predicate returns `false`. The arguments are in order, the current element, the tracked map flow, the length weight of the slice and the

[2]An orientation matrix can be obtained from the 3 angles with `W=matrix(3):rotzxy(-phi,theta,psi)`.

slice index. (default: `fnil`)
Example: `savesel = \e -> mylist[e.name] ~= nil`.

`savemap` A *logical* indicating to save the orientation matrix `W` in the column `__map` of the *mtable*. (default: `false`).
Example: `savemap = true`.

`atentry` A *callable* `(elm, mflw, 0, -1)` invoked at element entry. The arguments are in order, the current element, the tracked map flow, zero length and the slice index `-1`. (default: `fnil`).
Example: `atentry = myaction`.

`atslice` A *callable* `(elm, mflw, lw, islc)` invoked at element slice. The arguments are in order, the current element, the tracked map flow, the length weight of the slice and the slice index. (default: `fnil`).
Example: `atslice = myaction`.

`atexit` A *callable* `(elm, mflw, 0, -2)` invoked at element exit. The arguments are in order, the current element, the tracked map flow, zero length and the slice index `-2`. (default: `fnil`).
Example: `atexit = myaction`.

`atsave` A *callable* `(elm, mflw, lw, islc)` invoked at element saving steps, by default at exit. The arguments are in order, the current element, the tracked map flow, the length weight of the slice and the slice index. (default: `fnil`).
Example: `atsave = myaction`.

`atdebug` A *callable* `(elm, mflw, lw, [msg], [...])` invoked at the entry and exit of element maps during the integration steps, i.e. within the slices. The arguments are in order, the current element, the tracked map flow, the length weight of the integration step and a *string* specifying a debugging message, e.g. `"`*`map_name`*`:0"` for entry and `":1"` for exit. If the level `debug` $\geqslant 4$ and `atdebug` is not specified, the default *function* `mdump` is used. In some cases, extra arguments could be passed to the method. (default: `fnil`).
Example: `atdebug = myaction`.

`info` A *number* specifying the information level to control the verbosity of the output on the console. (default: `nil`).
Example: `info = 2`.

`debug` A *number* specifying the debug level to perform extra assertions and to control the verbosity of the output on the console. (default: `nil`).
Example: `debug = 2`.

`usrdef` Any user defined data that will be attached to the tracked map flow, which is internally passed to the elements method `:survey` and to their underlying maps. (default: `nil`).
Example: `usrdef = { myvar=somevalue }`.

`mflow` A *mflow* containing the current state of a `survey` command. If a map flow is provided, all attributes are discarded except `nstep`, `info` and `debug`, as the command was already set up upon its creation. (default: `nil`).
Example: `mflow = mflow0`.

The `survey` command returns the following objects in this order:

mtbl	A *mtable* corresponding to the TFS table of the `survey` command.
mflw	A *mflow* corresponding to the map flow of the `survey` command.
eidx	An optional *number* corresponding to the last surveyed element index in the sequence when `nstep` was specified and stopped the command before the end of the `range`.

2 Survey mtable

The `survey` command returns a *mtable* where the information described hereafter is the default list of fields written to the TFS files.[3]

The header of the *mtable* contains the fields in the default order:

name	The name of the command that created the *mtable*, e.g. `"survey"`.
type	The type of the *mtable*, i.e. `"survey"`.
title	The value of the command attribute `title`.
origin	The origin of the application that created the *mtable*, e.g. `"MAD 1.0.0 OSX 64"`.
date	The date of the creation of the *mtable*, e.g. `"27/05/20"`.
time	The time of the creation of the *mtable*, e.g. `"19:18:36"`.
refcol	The reference *column* for the *mtable* dictionnary, e.g. `"name"`.
direction	The value of the command attribute `dir`.
observe	The value of the command attribute `observe`.
implicit	The value of the command attribute `implicit`.
misalign	The value of the command attribute `misalign`.
range	The value of the command attribute `range`.[4]
__seq	The *sequence* from the command attribute `sequence`.[5]

The core of the *mtable* contains the columns in the default order:

name	The name of the element.
kind	The kind of the element.
s	The s-position at the end of the element slice.
l	The length from the start of the element to the end of the element slice.
angle	The angle from the start of the element to the end of the element slice.
tilt	The tilt of the element.
x	The global coordinate x at the s-position.

[3]The output of mtable in TFS files can be fully customized by the user.

[4]This field is not saved in the TFS table by default.

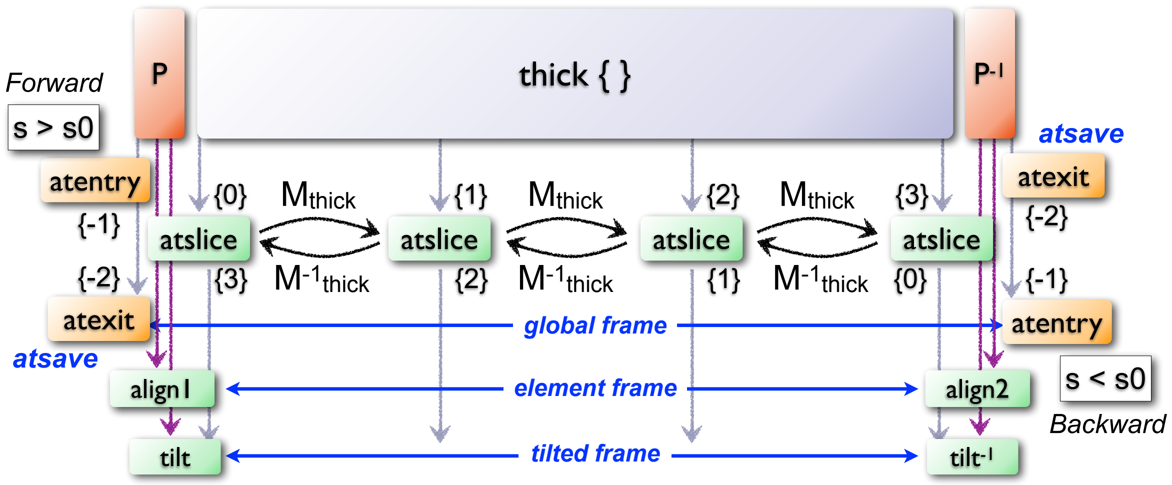
[5]Fields and columns starting with two underscores are protected data and never saved to TFS files.

`y`	The global coordinate y at the s-position.
`z`	The global coordinate z at the s-position.
`theta`	The global angle θ at the s-position.
`phi`	The global angle ϕ at the s-position.
`psi`	The global angle ψ at the s-position.
`slc`	The slice number ranging from `-2` to `nslice`.
`turn`	The turn number.
`tdir`	The t-direction of the tracking in the element.
`eidx`	The index of the element in the sequence.
`__map`	The orientation *matrix* at the s-position.[5]

3 Geometrical tracking

The Figure 11.2 presents the scheme of the geometrical tracking through an element sliced with `nslice=3`. The actions `atentry` (index `-1`), `atslice` (indexes `0`..3), and `atexit` (index `-2`) are reversed between the forward tracking (`dir=1` with increasing s-position) and the backward tracking (`dir=-1` with decreasing s-position). By default, the action `atsave` is attached to the exit slice, and hence it is also reversed in the backward tracking.

Figure 11.2: Geometrical tracking with slices.



3.1 Slicing

The slicing can take three different forms:

- A *number* of the form `nslice=`N that specifies the number of slices with indexes $0..N$. This defines a uniform slicing with slice length $l_{\text{slice}} = l_{\text{elem}}/N$.
- An *iterable* of the form `nslice={`lw_1`,`lw_2`,..,`lw_N`}` with $\sum_i lw_i = 1$ that specifies the fraction of length of each slice with indexes $0..N$ where N=`#nslice`. This defines a non-uniform slicing with a slice length of $l_i = lw_i \times l_{\text{elem}}$.

– A *callable* `(elm, mflw, lw)` returning one of the two previous forms of slicing. The arguments are in order, the current element, the tracked map flow, and the length weight of the step, which should allow to return a user-defined element-specific slicing.

The surrounding `P` and `P`$^{-1}$ maps represent the patches applied around the body of the element to change the frames, after the `atentry` and before the `atexit` actions:

– The misalignment of the element to move from the *global frame* to the *element frame* if the command attribute `misalign` is set to `true`.
– The tilt of the element to move from the element frame to the *titled frame* if the element attribute `tilt` is non-zero. The `atslice` actions take place in this frame.

These patches do not change the global frame per se, but they may affect the way that other components change the global frame, e.g. the tilt combined with the angle of a bending element.

3.2 Sub-elements

The `survey` command takes sub-elements into account, mainly for compatibility with the `track` command. In this case, the slicing specification is taken between sub-elements, e.g. 3 slices with 2 sub-elements gives a final count of 9 slices. It is possible to adjust the number of slices between sub-elements with the third form of slicing specifier, i.e. by using a callable where the length weight argument is between the current (or the end of the element) and the last sub-elements (or the start of the element).

4 Examples

TODO

Chapter 12. Track

The `track` command provides a simple interface to the *dynamic* tracking code.[1] The dynamic tracking can be used to track the particles in the local reference system while running through the elements of a sequence. The particles coordinates can be expressed in the global reference system by changing from the local to the global frames using the information delivered by the `survey` command.

1 Command synopsis

The `track` command format is summarized in Figure 12.1, including the default setup of the attributes. The `track` command supports the following attributes:

`sequence` The *sequence* to track. (no default, required).
Example: `sequence = lhcb1`.

`beam` The reference *beam* for the tracking. If no beam is provided, the command looks for a beam attached to the sequence, i.e. the attribute `seq.beam`.[2] (default: `nil`).
Example: `beam = beam 'lhcbeam' {` *`beam-attributes`* `}`.

`range` A *range* specifying the span of the sequence track. If no range is provided, the command looks for a range attached to the sequence, i.e. the attribute `seq.range`. (default: `nil`).
Example: `range = "S.DS.L8.B1/E.DS.R8.B1"`.

`dir` The s-direction of the tracking: `1` forward, `-1` backward. (default: `1`).
Example: `dir = -1`.

`s0` A *number* specifying the initial s-position offset. (default: `0` [m]).
Example: `s0 = 5000`.

`X0` A *mappable* (or a list of *mappable*) specifying initial coordinates `{x,px,y,py,t,pt}`, damap, or beta block for each tracked object, i.e. particle or damap. The beta blocks are converted to damaps, while the coordinates are converted to damaps only if `mapdef` is specified, but both will use `mapdef` to setup the damap constructor. Each tracked object may also contain a `beam` to override the reference beam, and a *logical* `nosave` to discard this object from being saved in the mtable. (default: `0`).
Example: `X0 = { x=1e-3, px=-1e-5 }`.

`O0` A *mappable* specifying initial coordinates `{x,px,y,py,t,pt}` of the reference orbit around which X0 definitions take place. If it has the attribute `cofind == true`, it will be used as an initial guess to search for the reference closed orbit. (default: `0`).
Example: `O0 = { x=1e-4, px=-2e-5, y=-2e-4, py=1e-5 }`.

`deltap` A *number* (or list of *number*) specifying the initial δ_p to convert (using the beam) and add to the `pt` of each tracked particle or damap. (default: `nil`).
Example: `s0 = 5000`.

`nturn` A *number* specifying the number of turn to track. (default: `1`).
Example: `nturn = 2`.

[1] MAD-NG implements only two tracking codes denominated the *geometric* and the *dynamic* tracking.
[2] Initial coordinates `X0` may override it by providing per particle or damap beam.

Figure 12.1: Synopsis of the `track` command with default setup.

```
mtbl, mflw [, eidx] = track {
  sequence=sequ,   -- sequence (required)
  beam=nil,        -- beam (or sequence.beam, required)
  range=nil,       -- range of tracking (or sequence.range)
  dir=1,           -- s-direction of tracking (1 or -1)
  s0=0,            -- initial s-position offset [m]
  X0=0,            -- initial coordinates (or damap(s), or beta block(s))
  O0=0,            -- initial coordinates of reference orbit
  deltap=nil,      -- initial deltap(s)
  nturn=1,         -- number of turns to track
  nstep=-1,        -- number of elements to track
  nslice=1,        -- number of slices (or weights) for each element
  mapdef=false,    -- setup for damap (or list of, true => {})
  method=2,        -- method or order for integration (1 to 8)
  model='TKT',     -- model for integration ('DKD' or 'TKT')
  ptcmodel=nil,    -- use strict PTC thick model (override option)
  implicit=false,  -- slice implicit elements too (e.g.  plots)
  misalign=false,  -- consider misalignment
  fringe=true,     -- enable fringe fields (see element.flags.fringe)
  radiate=false,   -- radiate at slices
  totalpath=false, -- variable 't' is the totalpath
  save=true,       -- create mtable and save results
  title=nil,       -- title of mtable (default seq.name)
  observe=1,       -- save only in observed elements (every n turns)
  savesel=fnil,    -- save selector (predicate)
  savemap=false,   -- save damap in the column __map
  atentry=fnil,    -- action called when entering an element
  atslice=fnil,    -- action called after each element slices
  atexit=fnil,     -- action called when exiting an element
  ataper=fnil,     -- action called when checking for aperture
  atsave=fnil,     -- action called when saving in mtable
  atdebug=fnil,    -- action called when debugging the element maps
  info=nil,        -- information level (output on terminal)
  debug=nil,       -- debug information level (output on terminal)
  usrdef=nil,      -- user defined data attached to the mflow
  mflow=nil,       -- mflow, exclusive with other attributes except nstep
}
```

`nstep` A *number* specifying the number of element to track. A negative value will track all elements. (default: `-1`).
Example: `nstep = 1`.

`nslice` A *number* specifying the number of slices or an *iterable* of increasing relative positions or a *callable* `(elm, mflw, lw)` returning one of the two previous kind of positions to track in the elements. The arguments of the callable are in order, the current element, the tracked map flow, and the length weight of the step. This attribute can be locally

overridden by the element. (default: `1`).
Example: `nslice = 5`.

`mapdef` A *logical* or a *damap* specification as defined by the DAmap module to track DA maps instead of particles coordinates. A value of `true` is equivalent to invoke the *damap* constructor with `{}` as argument. This attribute allows to track DA maps instead of particles. (default: `nil`).
Example: `mapdef = { xy=2, pt=5 }`.

`method` A *number* specifying the order of integration from 1 to 8, or a *string* specifying a special method of integration. Odd orders are rounded to the next even order to select the corresponding Yoshida or Boole integration schemes. The special methods are `simple` (equiv. to `DKD` order 2), `collim` (equiv. to `MKM` order 2), and `teapot` (Teapot splitting order 2). (default: `2`).
Example: `method = 'teapot'`.

`model` A *string* specifying the integration model, either `'DKD'` for *Drift-Kick-Drift* thin lens integration or `'TKT'` for *Thick-Kick-Thick* thick lens integration.[3] (default: `'TKT'`)
Example: `model = 'DKD'`.

`ptcmodel` A *logical* indicating to use strict PTC model.[4] (default: `nil`)
Example: `ptcmodel = true`.

`implicit` A *logical* indicating that implicit elements must be sliced too, e.g. for smooth plotting. (default: `false`).
Example: `implicit = true`.

`misalign` A *logical* indicating that misalignment must be considered. (default: `false`).
Example: `misalign = true`.

`fringe` A *logical* indicating that fringe fields must be considered or a *number* specifying a bit mask to apply to all elements fringe flags defined by the element module. The value `true` is equivalent to the bit mask `-1`, i.e. allow all elements (default) fringe fields. (default: `true`).
Example: `fringe = false`.

`radiate` A *logical* enabling or disabling the radiation or a *string* specifying the type of radiation: `'average'` or `'quantum'`. The value `true` is equivalent to `'average'`. The value `'quantum+photon'` enables the tracking of emitted photons. (default: `false`).
Example: `radiate = 'quantum'`.

`totalpath` A *logical* indicating to use the totalpath for the fifth variable `'t'` instead of the local path. (default: `false`).
Example: `totalpath = true`.

`save` A *logical* specifying to create a *mtable* and record tracking information at the observation points. The `save` attribute can also be a *string* specifying saving positions in the observed elements: `"atentry"`, `"atslice"`, `"atexit"` (i.e. `true`), `"atbound"` (i.e. entry and exit), `"atbody"` (i.e. slices and exit) and `"atall"`. (default: `true`).
Example: `save = false`.

[3] The `TKT` scheme (Yoshida) is automatically converted to the `MKM` scheme (Boole) when approriate.

[4] In all cases, MAD-NG uses PTC setup `time=true, exact=true`.

`title` A *string* specifying the title of the *mtable*. If no title is provided, the command looks for the name of the sequence, i.e. the attribute `seq.name`. (default: `nil`).
Example: `title = "track around IP5"`.

`observe` A *number* specifying the observation points to consider for recording the tracking information. A zero value will consider all elements, while a positive value will consider selected elements only, checked with method `:is_observed`, every `observe`>0 turns. (default: `1`).
Example: `observe = 1`.

`savesel` A *callable* `(elm, mflw, lw, islc)` acting as a predicate on selected elements for observation, i.e. the element is discarded if the predicate returns `false`. The arguments are in order, the current element, the tracked map flow, the length weight of the slice and the slice index. (default: `fnil`)
Example: `savesel = \e -> mylist[e.name] ~= nil`.

`savemap` A *logical* indicating to save the damap in the column `__map` of the *mtable*. (default: `false`).
Example: `savemap = true`.

`atentry` A *callable* `(elm, mflw, 0, -1)` invoked at element entry. The arguments are in order, the current element, the tracked map flow, zero length and the slice index `-1`. (default: `fnil`).
Example: `atentry = myaction`.

`atslice` A *callable* `(elm, mflw, lw, islc)` invoked at element slice. The arguments are in order, the current element, the tracked map flow, the length weight of the slice and the slice index. (default: `fnil`).
Example: `atslice = myaction`.

`atexit` A *callable* `(elm, mflw, 0, -2)` invoked at element exit. The arguments are in order, the current element, the tracked map flow, zero length and the slice index `-2`. (default: `fnil`).
Example: `atexit = myaction`.

`ataper` A *callable* `(elm, mflw, lw, islc)` invoked at element aperture checks, by default at last slice. The arguments are in order, the current element, the tracked map flow, the length weight of the slice and the slice index. If a particle or a damap hits the aperture, then its `status = "lost"` and it is removed from the list of tracked items. (default: `fnil`).
Example: `ataper = myaction`.

`atsave` A *callable* `(elm, mflw, lw, islc)` invoked at element saving steps, by default at exit. The arguments are in order, the current element, the tracked map flow, the length weight of the slice and the slice index. (default: `fnil`).
Example: `atsave = myaction`.

`atdebug` A *callable* `(elm, mflw, lw, [msg], [...])` invoked at the entry and exit of element maps during the integration steps, i.e. within the slices. The arguments are in order, the current element, the tracked map flow, the length weight of the integration step and a *string* specifying a debugging message, e.g. `"`*`map_name`*`:0"` for entry and `":1"` for exit. If the level `debug` $\geqslant 4$ and `atdebug` is not specified, the default *function* `mdump` is used. In some cases, extra arguments could be passed to the method. (default: `fnil`).
Example: `atdebug = myaction`.

`info`	A *number* specifying the information level to control the verbosity of the output on the console. (default: `nil`). Example: `info = 2`.
`debug`	A *number* specifying the debug level to perform extra assertions and to control the verbosity of the output on the console. (default: `nil`). Example: `debug = 2`.
`usrdef`	Any user defined data that will be attached to the tracked map flow, which is internally passed to the elements method `:track` and to their underlying maps. (default: `nil`). Example: `usrdef = { myvar=somevalue }`.
`mflow`	A *mflow* containing the current state of a `track` command. If a map flow is provided, all attributes are discarded except `nstep`, `info` and `debug`, as the command was already set up upon its creation. (default: `nil`). Example: `mflow = mflow0`.

The `track` command returns the following objects in this order:

`mtbl`	A *mtable* corresponding to the TFS table of the `track` command.
`mflw`	A *mflow* corresponding to the map flow of the `track` command.
`eidx`	An optional *number* corresponding to the last tracked element index in the sequence when `nstep` was specified and stopped the command before the end of the `range`.

2 Track mtable

The `track` command returns a *mtable* where the information described hereafter is the default list of fields written to the TFS files.[5]

The header of the *mtable* contains the fields in the default order:

`name`	The name of the command that created the *mtable*, e.g. `"track"`.
`type`	The type of the *mtable*, i.e. `"track"`.
`title`	The value of the command attribute `title`.
`origin`	The origin of the application that created the *mtable*, e.g. `"MAD 1.0.0 OSX 64"`.
`date`	The date of the creation of the *mtable*, e.g. `"27/05/20"`.
`time`	The time of the creation of the *mtable*, e.g. `"19:18:36"`.
`refcol`	The reference *column* for the *mtable* dictionnary, e.g. `"name"`.
`direction`	The value of the command attribute `dir`.
`observe`	The value of the command attribute `observe`.
`implicit`	The value of the command attribute `implicit`.
`misalign`	The value of the command attribute `misalign`.

[5]The output of mtable in TFS files can be fully customized by the user.

deltap	The value of the command attribute `deltap`.
lost	The number of lost particle(s) or damap(s).
range	The value of the command attribute `range`.[6]
__seq	The *sequence* from the command attribute `sequence`.[7]

The core of the *mtable* contains the columns in the default order:

name	The name of the element.
kind	The kind of the element.
s	The s-position at the end of the element slice.
l	The length from the start of the element to the end of the element slice.
id	The index of the particle or damap as provided in `X0`.
x	The local coordinate x at the s-position.
px	The local coordinate p_x at the s-position.
y	The local coordinate y at the s-position.
py	The local coordinate p_y at the s-position.
t	The local coordinate t at the s-position.
pt	The local coordinate p_t at the s-position.
pc	The reference beam P_0c in which p_t is expressed.
slc	The slice index ranging from `-2` to `nslice`.
turn	The turn number.
tdir	The t-direction of the tracking in the element.
eidx	The index of the element in the sequence.
status	The status of the particle or damap.
__map	The damap at the s-position.[3]

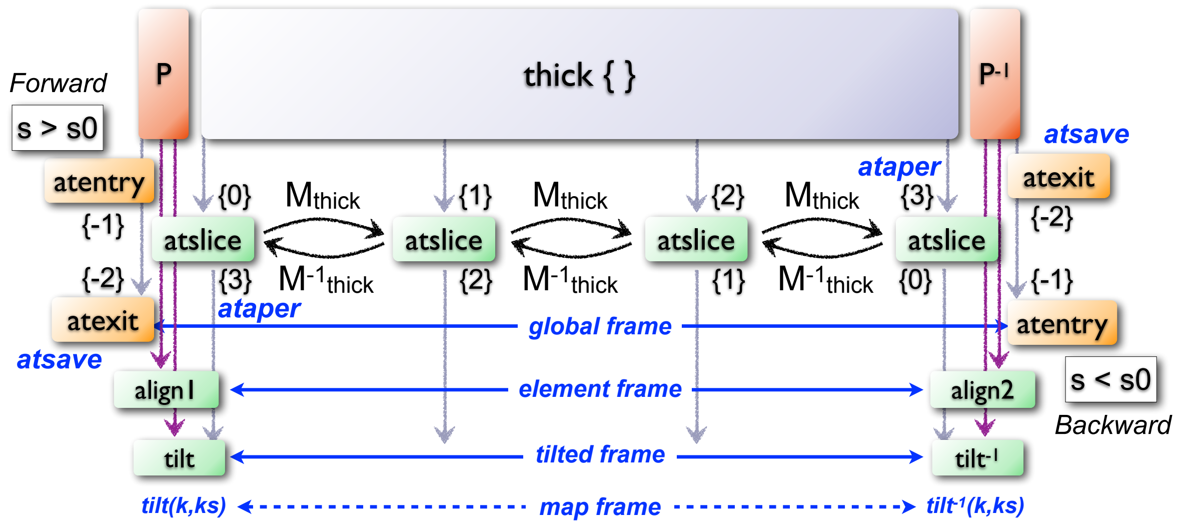
3 Dynamical tracking

The Figure 12.2 presents the scheme of the dynamical tracking through an element sliced with `nslice=3`. The actions `atentry` (index `-1`), `atslice` (indexes `0..3`), and `atexit` (index `-2`) are reversed between the forward tracking (`dir=1` with increasing s-position) and the backward tracking (`dir=-1` with decreasing s-position). By default, the action `atsave` is attached to the exit slice and the action `ataper` is attached to the last slice just before exit, i.e. to the last `atslice` action in the tilted frame, and hence they are also both reversed in the backward tracking.

[6]This field is not saved in the TFS table by default.

[7]Fields and columns starting with two underscores are protected data and never saved to TFS files.

Figure 12.2: Dynamical tracking with slices.



3.1 Slicing

The slicing can take three different forms:

- A *number* of the form `nslice=`N that specifies the number of slices with indexes $0..N$. This defines a uniform slicing with slice length $l_{\text{slice}} = l_{\text{elem}}/N$.
- An *iterable* of the form `nslice={`lw_1`,`lw_2`,..,`lw_N`}` with $\sum_i lw_i = 1$ that specifies the fraction of length of each slice with indexes $0..N$ where N`=#nslice`. This defines a non-uniform slicing with a slice length of $l_i = lw_i \times l_{\text{elem}}$.
- A *callable* `(elm, mflw, lw)` returning one of the two previous forms of slicing. The arguments are in order, the current element, the tracked map flow, and the length weight of the step, which should allow to return a user-defined element-specific slicing.

The surrounding `P` and `P`$^{-1}$ maps represent the patches applied around the body of the element to change the frames, after the `atentry` and before the `atexit` actions:

- The misalignment of the element to move from the *global frame* to the *element frame* if the command attribute `misalign` is set to `true`.
- The tilt of the element to move from the element frame to the *titled frame* if the element attribute `tilt` is non-zero. The `atslice` actions take place in this frame.

The *map frame* is specific to some maps while tracking through the body of the element. In principle, the map frame is not visible to the user, only to the integrator. For example, a quadrupole with both `k1` and `k1s` defined will have a *map frame* tilted by the angle $\alpha = -\frac{1}{2}\tan^{-1}\frac{k1s}{k1}$ attached to its thick map, i.e. the focusing matrix handling only $\tilde{k}_1 = \sqrt{k1^2 + k1s^2}$, but not to its thin map, i.e. the kick from all multipoles (minus `k1` and `k1s`) expressed in the *tilted frame*, during the integration steps.

3.2 Sub-elements

The `track` command takes sub-elements into account. In this case, the slicing specification is taken between sub-elements, e.g. 3 slices with 2 sub-elements gives a final count of 9 slices. It is possible to adjust the number of slices between sub-elements with the third form of slicing specifier, i.e. by using a callable where the length weight argument is between the current (or the end of the element) and the last sub-elements (or the start of the element).

3.3 Particles status

The `track` command initializes the map flow with particles or damaps or both, depending on the attributes `X0` and `mapdef`. The `status` attribute of each particle or damap will be set to one of `"Xset"`, `"Mset"`, and `"Aset"` to track the origin of its initialization: coordinates, damap, or normalizing damap (normal form or beta block). After the tracking, some particles or damaps may have the status `"lost"` and their number being recorded in the counter `lost` from TFS table header. Other commands like `cofind` or `twiss` may add extra tags to the status value, like `"stable"`, `"unstable"` and `"singular"`.

4 Examples

TODO

Chapter 13. Cofind

The `cofind` command (i.e. closed orbit finder) provides a simple interface to find a closed orbit using the Newton algorithm on top of the `track` command.

1 Command synopsis

The `cofind` command format is summarized in Figure 13.1, including the default setup of the attributes. Most of these attributes are set to `nil` by default, meaning that `cofind` relies on the `track` command defaults. The `cofind` command supports the following attributes:

`sequence`	The *sequence* to track. (no default, required). Example: `sequence = lhcb1`.
`beam`	The reference *beam* for the tracking. If no beam is provided, the command looks for a beam attached to the sequence, i.e. the attribute `seq.beam`.[1] (default: `nil`). Example: `beam = beam 'lhcbeam' {` *`beam-attributes`* `}`.
`range`	A *range* specifying the span of the sequence track. If no range is provided, the command looks for a range attached to the sequence, i.e. the attribute `seq.range`. (default: `nil`). Example: `range = "S.DS.L8.B1/E.DS.R8.B1"`.
`dir`	The s-direction of the tracking: `1` forward, `-1` backward. (default: `nil`). Example: `dir = -1`.
`s0`	A *number* specifying the initial s-position offset. (default: `nil`). Example: `s0 = 5000`.
`X0`	A *mappable* (or a list of *mappable*) specifying initial coordinates `{x,px,y,py, t,pt}`, damap, or beta block for each tracked object, i.e. particle or damap. The beta blocks are converted to damaps, while the coordinates are converted to damaps only if `mapdef` is specified, but both will use `mapdef` to setup the damap constructor. Each tracked object may also contain a `beam` to override the reference beam, and a *logical* `nosave` to discard this object from being saved in the mtable. (default: `nil`). Example: `X0 = { x=1e-3, px=-1e-5 }`.
`O0`	A *mappable* specifying initial coordinates `{x,px,y,py,t,pt}` of the reference orbit around which X0 definitions take place. If it has the attribute `cofind == true`, it will be used as an initial guess to search for the reference closed orbit. (default: `0`). Example: `O0 = { x=1e-4, px=-2e-5, y=-2e-4, py=1e-5 }`.
`deltap`	A *number* (or list of *number*) specifying the initial δ_p to convert (using the beam) and add to the `pt` of each tracked particle or damap. (default:`nil`). Example: `s0 = 5000`.
`nturn`	A *number* specifying the number of turn to track. (default: `nil`). Example: `nturn = 2`.
`nstep`	A *number* specifying the number of element to track. A negative value will track all elements. (default: `nil`). Example: `nstep = 1`.

[1] Initial coordinates `X0` may override it by providing a beam per particle or damap.

Figure 13.1: Synopsis of the `cofind` command with default setup.

```
mtbl, mflw = cofind {
  sequence=sequ,     -- sequence (required)
  beam=nil,          -- beam (or sequence.beam, required)
  range=nil,         -- range of tracking (or sequence.range)
  dir=nil,           -- s-direction of tracking (1 or -1)
  s0=nil,            -- initial s-position offset [m]
  X0=nil,            -- initial coordinates (or damap, or beta block)
  O0=nil,            -- initial coordinates of reference orbit
  deltap=nil,        -- initial deltap(s)
  nturn=nil,         -- number of turns to track
  nslice=nil,        -- number of slices (or weights) for each element
  mapdef=true,       -- setup for damap (or list of, true => {})
  method=nil,        -- method or order for integration (1 to 8)
  model=nil,         -- model for integration ('DKD' or 'TKT')
  ptcmodel=nil,      -- use strict PTC thick model (override option)
  implicit=nil,      -- slice implicit elements too (e.g.  plots)
  misalign=nil,      -- consider misalignment
  fringe=nil,        -- enable fringe fields (see element.flags.fringe)
  radiate=nil,       -- radiate at slices
  totalpath=nil,     -- variable 't' is the totalpath
  save=false,        -- create mtable and save results
  title=nil,         -- title of mtable (default seq.name)
  observe=nil,       -- save only in observed elements (every n turns)
  savesel=nil,       -- save selector (predicate)
  savemap=nil,       -- save damap in the column __map
  atentry=nil,       -- action called when entering an element
  atslice=nil,       -- action called after each element slices
  atexit=nil,        -- action called when exiting an element
  ataper=nil,        -- action called when checking for aperture
  atsave=nil,        -- action called when saving in mtable
  atdebug=fnil,      -- action called when debugging the element maps
  codiff=1e-10,      -- finite differences step for jacobian
  coiter=20,         -- maximum number of iterations
  cotol=1e-8,        -- closed orbit tolerance (i.e. |dX|)
  X1=0,              -- optional final coordinates translation
  info=nil,          -- information level (output on terminal)
  debug=nil,         -- debug information level (output on terminal)
  usrdef=nil,        -- user defined data attached to the mflow
  mflow=nil,         -- mflow, exclusive with other attributes
}
```

`nslice` A *number* specifying the number of slices or an *iterable* of increasing relative positions or a *callable* `(elm, mflw, lw)` returning one of the two previous kind of positions to track in the elements. The arguments of the callable are in order, the current element, the tracked map flow, and the length weight of the step. This attribute can be locally overridden by the element. (default: `nil`).

Example: `nslice = 5`.

`mapdef` A *logical* or a *damap* specification as defined by the DAmap module to track DA maps instead of particles coordinates. A value of `true` is equivalent to invoke the *damap* constructor with `{}` as argument. A value of `false` or `nil` disable the use of damaps and force `cofind` to replace each particles or damaps by seven particles to approximate their Jacobian by finite difference. (default: `true`).
Example: `mapdef = { xy=2, pt=5 }`.

`method` A *number* specifying the order of integration from 1 to 8, or a *string* specifying a special method of integration. Odd orders are rounded to the next even order to select the corresponding Yoshida or Boole integration schemes. The special methods are `simple` (equiv. to `DKD` order 2), `collim` (equiv. to `MKM` order 2), and `teapot` (Teapot splitting order 2). (default: `nil`).
Example: `method = 'teapot'`.

`model` A *string* specifying the integration model, either `'DKD'` for *Drift-Kick-Drift* thin lens integration or `'TKT'` for *Thick-Kick-Thick* thick lens integration.[2] (default: `nil`)
Example: `model = 'DKD'`.

`ptcmodel` A *logical* indicating to use strict PTC model.[3] (default: `nil`)
Example: `ptcmodel = true`.

`implicit` A *logical* indicating that implicit elements must be sliced too, e.g. for smooth plotting. (default: `nil`).
Example: `implicit = true`.

`misalign` A *logical* indicating that misalignment must be considered. (default: `nil`).
Example: `misalign = true`.

`fringe` A *logical* indicating that fringe fields must be considered or a *number* specifying a bit mask to apply to all elements fringe flags defined by the element module. The value `true` is equivalent to the bit mask `-1`, i.e. allow all elements (default) fringe fields. (default: `nil`).
Example: `fringe = false`.

`radiate` A *logical* enabling or disabling the radiation or the *string* specifying the `'average'` type of radiation. The value `true` is equivalent to `'average'` and the value `'quantum'` is converted to `'average'`. (default: `nil`).
Example: `radiate = 'average'`.

`totalpath` A *logical* indicating to use the totalpath for the fifth variable `'t'` instead of the local path. (default: `nil`).
Example: `totalpath = true`.

`save` A *logical* specifying to create a *mtable* and record tracking information at the observation points. The `save` attribute can also be a *string* specifying saving positions in the observed elements: `"atentry"`, `"atslice"`, `"atexit"` (i.e. `true`), `"atbound"` (i.e. entry and exit), `"atbody"` (i.e. slices and exit) and `"atall"`. (default: `false`).
Example: `save = false`.

[2] The `TKT` scheme (Yoshida) is automatically converted to the `MKM` scheme (Boole) when approriate.
[3] In all cases, MAD-NG uses PTC setup `time=true, exact=true`.

`title` A *string* specifying the title of the *mtable*. If no title is provided, the command looks for the name of the sequence, i.e. the attribute `seq.name`. (default: `nil`).
Example: `title = "track around IP5"`.

`observe` A *number* specifying the observation points to consider for recording the tracking information. A zero value will consider all elements, while a positive value will consider selected elements only, checked with method `:is_observed`, every `observe`>0 turns. (default: `nil`).
Example: `observe = 1`.

`savesel` A *callable* `(elm, mflw, lw, islc)` acting as a predicate on selected elements for observation, i.e. the element is discarded if the predicate returns `false`. The arguments are in order, the current element, the tracked map flow, the length weight of the slice and the slice index. (default: `nil`)
Example: `savesel = \e -> mylist[e.name] ~= nil`.

`savemap` A *logical* indicating to save the damap in the column `__map` of the *mtable*. (default: `nil`).
Example: `savemap = true`.

`atentry` A *callable* `(elm, mflw, 0, -1)` invoked at element entry. The arguments are in order, the current element, the tracked map flow, zero length and the slice index `-1`. (default: `nil`).
Example: `atentry = myaction`.

`atslice` A *callable* `(elm, mflw, lw, islc)` invoked at element slice. The arguments are in order, the current element, the tracked map flow, the length weight of the slice and the slice index. (default: `nil`).
Example: `atslice = myaction`.

`atexit` A *callable* `(elm, mflw, 0, -2)` invoked at element exit. The arguments are in order, the current element, the tracked map flow, zero length and the slice index `-2`. (default: `nil`).
Example: `atexit = myaction`.

`ataper` A *callable* `(elm, mflw, lw, islc)` invoked at element aperture checks, by default at last slice. The arguments are in order, the current element, the tracked map flow, the length weight of the slice and the slice index. If a particle or a damap hits the aperture, then its `status = "lost"` and it is removed from the list of tracked items. (default: `fnil`).
Example: `ataper = myaction`.

`atsave` A *callable* `(elm, mflw, lw, islc)` invoked at element saving steps, by default at exit. The arguments are in order, the current element, the tracked map flow, the length weight of the slice and the slice index. (default: `nil`).
Example: `atsave = myaction`.

`atdebug` A *callable* `(elm, mflw, lw, [msg], [...])` invoked at the entry and exit of element maps during the integration steps, i.e. within the slices. The arguments are in order, the current element, the tracked map flow, the length weight of the integration step and a *string* specifying a debugging message, e.g. `"`*`map_name`*`:0"` for entry and `":1"` for exit. If the level `debug` $\geqslant 4$ and `atdebug` is not specified, the default *function* `mdump` is used. In some cases, extra arguments could be passed to the method. (default: `fnil`).
Example: `atdebug = myaction`.

codiff A *number* specifying the finite difference step to approximate the Jacobian when damaps are disabled. If `codiff` is larger than $100\times$`cotol`, it will be adjusted to `cotol`$/100$ and a warning will be emitted. (default: `1e-10`).
Example: `codiff = 1e-8`.

coiter A *number* specifying the maximum number of iteration. If this threshold is reached, all the remaining tracked objects are tagged as `"unstable"`. (default: `20`).
Example: `coiter = 5`.

cotol A *number* specifying the closed orbit tolerance. If all coordinates update of a particle or a damap are smaller than `cotol`, then it is tagged as `"stable"`. (default: `1e-8`).
Example: `cotol = 1e-6`.

X1 A *mappable* specifying the coordinates `{x,px,y,py,t,pt}` to *subtract* to the final coordinates of the particles or the damaps. (default: `0`).
Example: `X1 = { t=100, pt=10 }`.

info A *number* specifying the information level to control the verbosity of the output on the console. (default: `nil`).
Example: `info = 2`.

debug A *number* specifying the debug level to perform extra assertions and to control the verbosity of the output on the console. (default: `nil`).
Example: `debug = 2`.

usrdef Any user defined data that will be attached to the tracked map flow, which is internally passed to the elements method `:track` and to their underlying maps. (default: `nil`).
Example: `usrdef = { myvar=somevalue }`.

mflow A *mflow* containing the current state of a `track` command. If a map flow is provided, all attributes are discarded except `nstep`, `info` and `debug`, as the command was already set up upon its creation. (default: `nil`).
Example: `mflow = mflow0`.

The `cofind` command stops when all particles or damap are tagged as `"stable"`, `"unstable"`, `"singular"` or `"lost"`. The `cofind` command returns the following objects in this order:

mtbl A *mtable* corresponding to the TFS table of the `track` command where the `status` column may also contain the new values `"stable"`, `"unstable"` or `"singular"`.

mflw A *mflow* corresponding to the map flow of the `track` command. The particles or damaps `status` are tagged and ordered by `"stable"`, `"unstable"`, `"singular"`, `"lost"` and `id`.

2 Cofind mtable

The `cofind` command returns the `track` *mtable* unmodified except for the `status` column. The tracked objects id will appear once per iteration at the `$end` marker, and other defined observation points if any, until they are removed from the list of tracked objects.

3 Examples

TODO

Chapter 14. Twiss

The `twiss` command provides a simple interface to compute the optical functions around an orbit on top of the `track` command, and the `cofind` command if the search for closed orbits is requested.

1 Command synopsis

The `twiss` command format is summarized in Figure 14.1, including the default setup of the attributes. Most of these attributes are set to `nil` by default, meaning that `twiss` relies on the `track` and the `cofind` commands defaults. The `twiss` command supports the following attributes:

`sequence` The *sequence* to track. (no default, required).
Example: `sequence = lhcb1`.

`beam` The reference *beam* for the tracking. If no beam is provided, the command looks for a beam attached to the sequence, i.e. the attribute `seq.beam`.[1] (default: `nil`).
Example: `beam = beam 'lhcbeam' {` *`beam-attributes`* `}`.

`range` A *range* specifying the span of the sequence track. If no range is provided, the command looks for a range attached to the sequence, i.e. the attribute `seq.range`. (default: `nil`).
Example: `range = "S.DS.L8.B1/E.DS.R8.B1"`.

`dir` The s-direction of the tracking: `1` forward, `-1` backward. (default: `nil`).
Example: `dir = -1`.

`s0` A *number* specifying the initial s-position offset. (default: `nil`).
Example: `s0 = 5000`.

`X0` A *mappable* (or a list of *mappable*) specifying initial coordinates `{x,px,y,py, t,pt}`, damap, or beta0 block for each tracked object, i.e. particle or damap. The beta0 blocks are converted to damaps, while the coordinates are converted to damaps only if `mapdef` is specified, but both will use `mapdef` to setup the damap constructor. A closed orbit will be automatically searched for damaps built from coordinates. Each tracked object may also contain a `beam` to override the reference beam, and a *logical* `nosave` to discard this object from being saved in the mtable. (default: `0`).
Example: `X0 = { x=1e-3, px=-1e-5 }`.

`O0` A *mappable* specifying initial coordinates `{x,px,y,py,t,pt}` of the reference orbit around which X0 definitions take place. If it has the attribute `cofind == true`, it will be used as an initial guess to search for the reference closed orbit. (default: `0`).
Example: `O0 = { x=1e-4, px=-2e-5, y=-2e-4, py=1e-5 }`.

`deltap` A *number* (or list of *number*) specifying the initial δ_p to convert (using the beam) and add to the `pt` of each tracked particle or damap. (default:`nil`).
Example: `s0 = 5000`.

`chrom` A *logical* specifying to calculate the chromatic functions by finite different using an extra $\delta_p =$ `1e-6`. (default: `false`).
Example: `chrom = true`.

[1] Initial coordinates `X0` may override it by providing a beam per particle or damap.

Figure 14.1: Synopsis of the twiss command with default setup.

```
mtbl, mflw [, eidx] = twiss {
  sequence=sequ,   -- sequence (required)
  beam=nil,        -- beam (or sequence.beam, required)
  range=nil,       -- range of tracking (or sequence.range)
  dir=nil,         -- s-direction of tracking (1 or -1)
  s0=nil,          -- initial s-position offset [m]
  X0=nil,          -- initial coordinates (or damap(s), or beta block(s))
  O0=nil,          -- initial coordinates of reference orbit
  deltap=nil,      -- initial deltap(s)
  chrom=false,     -- compute chromatic functions by finite difference
  coupling=false,  -- compute optical functions for non-diagonal modes
  nturn=nil,       -- number of turns to track
  nstep=nil,       -- number of elements to track
  nslice=nil,      -- number of slices (or weights) for each element
  mapdef=true,     -- setup for damap (or list of, true => {})
  method=nil,      -- method or order for integration (1 to 8)
  model=nil,       -- model for integration ('DKD' or 'TKT')
  ptcmodel=nil,    -- use strict PTC thick model (override option)
  implicit=nil,    -- slice implicit elements too (e.g.  plots)
  misalign=nil,    -- consider misalignment
  fringe=nil,      -- enable fringe fields (see element.flags.fringe)
  radiate=nil,     -- radiate at slices
  totalpath=nil,   -- variable 't' is the totalpath
  save=true,       -- create mtable and save results
  title=nil,       -- title of mtable (default seq.name)
  observe=0,       -- save only in observed elements (every n turns)
  savesel=nil,     -- save selector (predicate)
  savemap=nil,     -- save damap in the column __map
  atentry=nil,     -- action called when entering an element
  atslice=nil,     -- action called after each element slices
  atexit=nil,      -- action called when exiting an element
  ataper=nil,      -- action called when checking for aperture
  atsave=nil,      -- action called when saving in mtable
  atdebug=fnil,    -- action called when debugging the element maps
  codiff=nil,      -- finite differences step for jacobian
  coiter=nil,      -- maximum number of iterations
  cotol=nil,       -- closed orbit tolerance (i.e. |dX|)
  X1=nil,          -- optional final coordinates translation
  info=nil,        -- information level (output on terminal)
  debug=nil,       -- debug information level (output on terminal)
  usrdef=nil,      -- user defined data attached to the mflow
  mflow=nil,       -- mflow, exclusive with other attributes
}
```

coupling A *logical* specifying to calculate the optical functions for coupling terms in the normalized forms. (default: false).

Example: `chrom = true`.

`nturn` A *number* specifying the number of turn to track. (default: `nil`).
Example: `nturn = 2`.

`nstep` A *number* specifying the number of element to track. A negative value will track all elements. (default: `nil`).
Example: `nstep = 1`.

`nslice` A *number* specifying the number of slices or an *iterable* of increasing relative positions or a *callable* `(elm, mflw, lw)` returning one of the two previous kind of positions to track in the elements. The arguments of the callable are in order, the current element, the tracked map flow, and the length weight of the step. This attribute can be locally overridden by the element. (default: `nil`).
Example: `nslice = 5`.

`mapdef` A *logical* or a *damap* specification as defined by the DAmap module to track DA maps instead of particles coordinates. A value of `true` is equivalent to invoke the *damap* constructor with `{}` as argument. A value of `false` or `nil` will be internally forced to `true` for the tracking of the normalized forms. (default: `true`).
Example: `mapdef = { xy=2, pt=5 }`.

`method` A *number* specifying the order of integration from 1 to 8, or a *string* specifying a special method of integration. Odd orders are rounded to the next even order to select the corresponding Yoshida or Boole integration schemes. The special methods are `simple` (equiv. to `DKD` order 2), `collim` (equiv. to `MKM` order 2), and `teapot` (Teapot splitting order 2). (default: `nil`).
Example: `method = 'teapot'`.

`model` A *string* specifying the integration model, either `'DKD'` for *Drift-Kick-Drift* thin lens integration or `'TKT'` for *Thick-Kick-Thick* thick lens integration.[2] (default: `nil`)
Example: `model = 'DKD'`.

`ptcmodel` A *logical* indicating to use strict PTC model.[3] (default: `nil`)
Example: `ptcmodel = true`.

`implicit` A *logical* indicating that implicit elements must be sliced too, e.g. for smooth plotting. (default: `nil`).
Example: `implicit = true`.

`misalign` A *logical* indicating that misalignment must be considered. (default: `nil`).
Example: `misalign = true`.

`fringe` A *logical* indicating that fringe fields must be considered or a *number* specifying a bit mask to apply to all elements fringe flags defined by the element module. The value `true` is equivalent to the bit mask `-1`, i.e. allow all elements (default) fringe fields. (default: `nil`).
Example: `fringe = false`.

`radiate` A *logical* enabling or disabling the radiation or the *string* specifying the `'average'` type of radiation during the closed orbit search. The value `true` is equivalent to `'average'`

[2]The `TKT` scheme (Yoshida) is automatically converted to the `MKM` scheme (Boole) when approriate.

[3]In all cases, MAD-NG uses PTC setup `time=true, exact=true`.

and the value `'quantum'` is converted to `'average'`. (default: `nil`).
Example: `radiate = 'average'`.

`totalpath` A *logical* indicating to use the totalpath for the fifth variable `'t'` instead of the local path. (default: `nil`).
Example: `totalpath = true`.

`save` A *logical* specifying to create a *mtable* and record tracking information at the observation points. The `save` attribute can also be a *string* specifying saving positions in the observed elements: `"atentry"`, `"atslice"`, `"atexit"` (i.e. `true`), `"atbound"` (i.e. entry and exit), `"atbody"` (i.e. slices and exit) and `"atall"`. (default: `false`).
Example: `save = false`.

`title` A *string* specifying the title of the *mtable*. If no title is provided, the command looks for the name of the sequence, i.e. the attribute `seq.name`. (default: `nil`).
Example: `title = "track around IP5"`.

`observe` A *number* specifying the observation points to consider for recording the tracking information. A zero value will consider all elements, while a positive value will consider selected elements only, checked with method `:is_observed`, every `observe`>0 turns. (default: `nil`).
Example: `observe = 1`.

`savesel` A *callable* `(elm, mflw, lw, islc)` acting as a predicate on selected elements for observation, i.e. the element is discarded if the predicate returns `false`. The arguments are in order, the current element, the tracked map flow, the length weight of the slice and the slice index. (default: `fnil`)
Example: `savesel = \e -> mylist[e.name] ~= nil`.

`savemap` A *logical* indicating to save the damap in the column `__map` of the *mtable*. (default: `nil`).
Example: `savemap = true`.

`atentry` A *callable* `(elm, mflw, 0, -1)` invoked at element entry. The arguments are in order, the current element, the tracked map flow, zero length and the slice index `-1`. (default: `fnil`).
Example: `atentry = myaction`.

`atslice` A *callable* `(elm, mflw, lw, islc)` invoked at element slice. The arguments are in order, the current element, the tracked map flow, the length weight of the slice and the slice index. (default: `fnil`).
Example: `atslice = myaction`.

`atexit` A *callable* `(elm, mflw, 0, -2)` invoked at element exit. The arguments are in order, the current element, the tracked map flow, zero length and the slice index `-2`. (default: `fnil`).
Example: `atexit = myaction`.

`ataper` A *callable* `(elm, mflw, lw, islc)` invoked at element aperture checks, by default at last slice. The arguments are in order, the current element, the tracked map flow, the length weight of the slice and the slice index. If a particle or a damap hits the aperture, then its `status = "lost"` and it is removed from the list of tracked items. (default: `fnil`).
Example: `ataper = myaction`.

`atsave` A *callable* `(elm, mflw, lw, islc)` invoked at element saving steps, by default at exit. The arguments are in order, the current element, the tracked map flow, the length weight of the slice and the slice index. (default: `fnil`).
Example: `atsave = myaction`.

`atdebug` A *callable* `(elm, mflw, lw, [msg], [...])` invoked at the entry and exit of element maps during the integration steps, i.e. within the slices. The arguments are in order, the current element, the tracked map flow, the length weight of the integration step and a *string* specifying a debugging message, e.g. `"`*`map_name`*`:0"` for entry and `":1"` for exit. If the level `debug` $\geqslant 4$ and `atdebug` is not specified, the default *function* `mdump` is used. In some cases, extra arguments could be passed to the method. (default: `fnil`).
Example: `atdebug = myaction`.

`codiff` A *number* specifying the finite difference step to approximate the Jacobian when damaps are disabled. If `codiff` is larger than $100\times$`cotol`, it will be adjusted to `cotol`$/100$ and a warning will be emitted. (default: `1e-10`).
Example: `codiff = 1e-8`.

`coiter` A *number* specifying the maximum number of iteration. If this threshold is reached, all the remaining tracked objects are tagged as `"unstable"`. (default: `20`).
Example: `coiter = 5`.

`cotol` A *number* specifying the closed orbit tolerance. If all coordinates update of a particle or a damap are smaller than `cotol`, then it is tagged as `"stable"`. (default: `1e-8`).
Example: `cotol = 1e-6`.

`X1` A *mappable* specifying the coordinates `{x,px,y,py,t,pt}` to *subtract* to the final coordinates of the particles or the damaps. (default: `0`).
Example: `X1 = { t=100, pt=10 }`.

`info` A *number* specifying the information level to control the verbosity of the output on the console. (default: `nil`).
Example: `info = 2`.

`debug` A *number* specifying the debug level to perform extra assertions and to control the verbosity of the output on the console. (default: `nil`).
Example: `debug = 2`.

`usrdef` Any user defined data that will be attached to the tracked map flow, which is internally passed to the elements method `:track` and to their underlying maps. (default: `nil`).
Example: `usrdef = { myvar=somevalue }`.

`mflow` A *mflow* containing the current state of a `track` command. If a map flow is provided, all attributes are discarded except `nstep`, `info` and `debug`, as the command was already set up upon its creation. (default: `nil`).
Example: `mflow = mflow0`.

The `twiss` command returns the following objects in this order:

`mtbl` A *mtable* corresponding to the augmented TFS table of the `track` command with the `twiss` command columns.

`mflw` A *mflow* corresponding to the augmented map flow of the `track` command with the `twiss` command data.

eidx	An optional *number* corresponding to the last tracked element index in the sequence when `nstep` was specified and stopped the command before the end of the `range`.

2 Twiss mtable

The `twiss` command returns a *mtable* where the information described hereafter is the default list of fields written to the TFS files.[4]

The header of the *mtable* contains the fields in the default order:[5]

name	The name of the command that created the *mtable*, e.g. `"track"`.
type	The type of the *mtable*, i.e. `"track"`.
title	The value of the command attribute `title`.
origin	The origin of the application that created the *mtable*, e.g. `"MAD 1.0.0 OSX 64"`.
date	The date of the creation of the *mtable*, e.g. `"27/05/20"`.
time	The time of the creation of the *mtable*, e.g. `"19:18:36"`.
refcol	The reference *column* for the *mtable* dictionnary, e.g. `"name"`.
direction	The value of the command attribute `dir`.
observe	The value of the command attribute `observe`.
implicit	The value of the command attribute `implicit`.
misalign	The value of the command attribute `misalign`.
deltap	The value of the command attribute `deltap`.
lost	The number of lost particle(s) or damap(s).
chrom	The value of the command attribute `chrom`.
coupling	The value of the command attribute `coupling`.
length	The s-length of the tracked design orbit.
q1	The tunes of mode 1.
q2	The tunes of mode 2.
q3	The tunes of mode 3.
alfap	The momentum compaction factor α_p.
etap	The phase slip factor η_p.
gammatr	The energy gamma transition γ_{tr}.
synch_1	The first synchroton radiation integral.
synch_2	The second synchroton radiation integral.

[4]The output of mtable in TFS files can be fully customized by the user.

[5]The fields from `name` to `lost` are set by the `track` command.

synch_3	The third synchroton radiation integral.
synch_4	The fourth synchroton radiation integral.
synch_5	The fifth synchroton radiation integral.
synch_6	The sixth synchroton radiation integral.
synch_8	The eighth synchroton radiation integral.
range	The value of the command attribute `range`.[6]
__seq	The *sequence* from the command attribute `sequence`.[7]

The core of the *mtable* contains the columns in the default order:[8]

name	The name of the element.
kind	The kind of the element.
s	The s-position at the end of the element slice.
l	The length from the start of the element to the end of the element slice.
id	The index of the particle or damap as provided in `X0`.
x	The local coordinate x at the s-position .
px	The local coordinate p_x at the s-position.
y	The local coordinate y at the s-position.
py	The local coordinate p_y at the s-position.
t	The local coordinate t at the s-position.
pt	The local coordinate p_t at the s-position.
slc	The slice index ranging from `-2` to `nslice`.
turn	The turn number.
tdir	The t-direction of the tracking in the element.
eidx	The index of the element in the sequence.
status	The status of the particle or damap.
alfa11	The optical function α of mode 1 at the s-position.
beta11	The optical function β of mode 1 at the s-position.
gama11	The optical function γ of mode 1 at the s-position.
mu1	The phase advance μ of mode 1 at the s-position.

[6]This field is not saved in the TFS table by default.
[7]Fields and columns starting with two underscores are protected data and never saved to TFS files.
[8]The column from `name` to `status` are set by the `track` command.

dx	The dispersion function of x at the s-position.
dpx	The dispersion function of p_x at the s-position.
alfa22	The optical function α of mode 2 at the s-position.
beta22	The optical function β of mode 2 at the s-position.
gama22	The optical function γ of mode 2 at the s-position.
mu2	The phase advance μ of mode 2 at the s-position.
dy	The dispersion function of y at the s-position.
dpy	The dispersion function of p_y at the s-position.
alfa33	The optical function α of mode 3 at the s-position.
beta33	The optical function β of mode 3 at the s-position.
gama33	The optical function γ of mode 3 at the s-position.
mu3	The phase advance μ of mode 3 at the s-position.
__map	The damap at the s-position.[7]

The `chrom` attribute will add the following fields to the *mtable* header:

dq1	The chromatic derivative of tunes of mode 1, i.e. chromaticities.
dq2	The chromatic derivative of tunes of mode 2, i.e. chromaticities.
dq3	The chromatic derivative of tunes of mode 3, i.e. chromaticities.

The `chrom` attribute will add the following columns to the *mtable*:

dmu1	The chromatic derivative of the phase advance of mode 1 at the s-position.
ddx	The chromatic derivative of the dispersion function of x at the s-position.
ddpx	The chromatic derivative of the dispersion function of p_x at the s-position.
wx	The chromatic amplitude function of mode 1 at the s-position.
phix	The chromatic phase function of mode 1 at the s-position.
dmu2	The chromatic derivative of the phase advance of mode 2 at the s-position.
ddy	The chromatic derivative of the dispersion function of y at the s-position.
ddpy	The chromatic derivative of the dispersion function of p_y at the s-position.
wy	The chromatic amplitude function of mode 2 at the s-position.
phiy	The chromatic phase function of mode 2 at the s-position.

The `coupling` attribute will add the following columns to the *mtable*:

alfa12	The optical function α of coupling mode 1-2 at the s-position.

beta12	The optical function β of coupling mode 1-2 at the s-position.
gama12	The optical function γ of coupling mode 1-2 at the s-position.
alfa13	The optical function α of coupling mode 1-3 at the s-position.
beta13	The optical function β of coupling mode 1-3 at the s-position.
gama13	The optical function γ of coupling mode 1-3 at the s-position.
alfa21	The optical function α of coupling mode 2-1 at the s-position.
beta21	The optical function β of coupling mode 2-1 at the s-position.
gama21	The optical function γ of coupling mode 2-1 at the s-position.
alfa23	The optical function α of coupling mode 2-3 at the s-position.
beta23	The optical function β of coupling mode 2-3 at the s-position.
gama23	The optical function γ of coupling mode 2-3 at the s-position.
alfa31	The optical function α of coupling mode 3-1 at the s-position.
beta31	The optical function β of coupling mode 3-1 at the s-position.
gama31	The optical function γ of coupling mode 3-1 at the s-position.
alfa32	The optical function α of coupling mode 3-2 at the s-position.
beta32	The optical function β of coupling mode 3-2 at the s-position.
gama32	The optical function γ of coupling mode 3-2 at the s-position.

3 Tracking linear normal form

TODO

4 Examples

TODO

Chapter 15. Match

The match command provides a unified interface to several optimizer. It can be used to match optics parameters (its main purpose), to fit data sets with parametric functions in the least-squares sense, or to find local or global minima of non-linear problems. Most local methods support bounds, equalities and inequalities constraints. The *least-squares* methods are custom variant of the Newton-Raphson and the Gauss-Newton algorithms implemented by the LSopt module. The local and global *non-linear* methods are relying on the NLopt module, which interfaces the embedded NLopt library that implements a dozen of well-known algorithms.

1 Command synopsis

Figure 15.1: Synopsis of the match command with default setup.

```
status, fmin, ncall = match {
  command      = function or nil,
  variables    = { variables-attributes,
                   { variable-attributes },
                   ...more variable definitions...
                   { variable-attributes } },
  equalities   = { constraints-attributes,
                   { constraint-attributes },
                   ...more equality definitions...
                   { constraint-attributes } },
  inequalities = { constraints-attributes,
                   { constraint-attributes },
                   ...more inequality definitions...
                   { constraint-attributes } },
  weights      = { weights-list },
  objective    = { objective-attributes },
  maxcall=nil, -- call limit
  maxtime=nil, -- time limit
  info=nil,    -- information level (output on terminal)
  debug=nil,   -- debug information level (output on terminal)
  usrdef=nil,  -- user defined data attached to the environment
}
```

The match command format is summarized in Figure 15.1, including the default setup of the attributes. The match command supports the following attributes:

command A *callable* (e) that will be invoked during the optimization process at each iteration. (default: nil).
Example: command := twiss { *twiss-attributes* }.

variables An *mappable* of single variable specification that can be combined with a *set* of specifications for all variables. (no default, required).
Example: variables = {{ var="seq.knobs.mq_k1" }}.

`equalities`	An *mappable* of single equality specification that can be combined with a *set* of specifications for all equalities. (default: `{}`). Example: `equalities = {{ expr=\t -> t.q1-64.295, name='q1' }}`.
`inequalities`	An *mappable* of single inequality specification that can be combined with a *set* of specifications for all inequalities. (default: `{}`). Example: `inequalities = {{ expr=\t -> t.mq4.beta11-50 }}`.
`weights`	A *mappable* of weights specification that can be used in the `kind` attribute of the constraints specifications. (default: `{}`). Example: `weights = { px=10 }`.
`objective`	A *mappable* of specifications for the objective to minimize. (default: `{}`). Example: `objective = { method="LD_LMDIF", fmin=1e-10 }`.
`maxcall`	A *number* specifying the maximum allowed calls of the `command` function or the `objective` function. (default: `nil`). Example: `maxcall = 100`.
`maxtime`	A *number* specifying the maximum allowed time in seconds. (default: `nil`). Example: `maxtime = 60`.
`info`	A *number* specifying the information level to control the verbosity of the output on the console. (default: `nil`). Example: `info = 3`.
`debug`	A *number* specifying the debug level to perform extra assertions and to control the verbosity of the output on the console. (default: `nil`). Example: `debug = 2`.
`usrdef`	Any user defined data that will be attached to the matching environment, which is passed as extra argument to all user defined functions in the `match` command. (default: `nil`). Example: `usrdef = { var=vector(15) }`.

The `match` command returns the following values in this order:

`status`	A *string* corresponding to the status of the command or the stopping reason of the method. See Table 15.1 for the list of supported status.
`fmin`	A *number* corresponding to the best minimum reached during the optimization.
`ncall`	The *number* of calls of the `command` function or the `objective` function.

2 Environment

The `match` command creates a matching environment, which is passed as argument to user's functions invoked during an iteration. It contains some useful attributes that can be read or changed during the optimization process (with care):

`ncall`	The current *number* of calls of the `command` and/or the `objective` functions.
`dtime`	A *number* reporting the current elapsed time.
`stop`	A *logical* stopping the `match` command immediately if set to `true`.

Table 15.1: List of `status` (*string*) returned by the `match` command.

`status`	Meaning
`SUCCESS`	Generic success (NLopt only, unlikely).
`FMIN`	`fmin` criteria is fulfilled by the objective function.
`FTOL`	`tol` or `rtol` criteria are fulfilled by the objective function.
`XTOL`	`tol` or `rtol` criteria are fulfilled by the variables step.
`MAXCALL`	`maxcall` criteria is reached.
`MAXTIME`	`maxtime` criteria is reached.
`ROUNDOFF`	Round off limited iteration progress, results may still be useful.
`STOPPED`	Termination forced by user, i.e. `env.stop = true`.
Errors	
`FAILURE`	Generic failure (NLopt only, unlikely).
`INVALID_ARGS`	Invalid argument (NLopt only, unlikely).
`OUT_OF_MEMORY`	Ran out of memory (NLopt only, unlikely).

`info`	The current information level $\geqslant 0$.
`debug`	The current debugging level $\geqslant 0$.
`usrdef`	The `usrdef` attribute of the `match` command or `nil`.
`command`	The `command` attribute of the `match` command or `nil`.
`variables`	The `variables` attribute of the `match` command.
`equalities`	The `equalities` attribute of the `match` command or `{}`.
`inequalities`	The `inequalities` attribute of the `match` command or `{}`.
`weights`	The `weights` attribute of the `match` command or `{}`.

3 Command

The attribute `command` (default: `nil`) must be a *callable* `(e)` that will be invoked with the matching environment as first argument during the optimization, right after the update of the variables to their new values, and before the evaluation of the constraints and the objective function. (default: `nil`).

```
command = function or nil,
```

The value returned by `command` is passed as the first argument to all constraints. If this return value is `nil`, the `match` command considers the current iteration as invalid. Depending on the selected method, the optimizer can start a new iteration or stop.
A typical `command` definition for matching optics is a function that calls a `twiss` command[1]:

```
command := mchklost( twiss { twiss-attributes } )
```

where the function `mchklost` surrounding the `twiss` command checks if the returned `mtable` (i.e. the twiss table) has lost particles and returns `nil` instead:

[1]Here, the function (i.e. the deferred expression) ignores the matching environment passed as first argument.

```
mchklost = \mt -> mt.lost == 0 and mt or nil
```

The function `mchklost`[2] is useful to avoid that all constraints do the check individually.

4 Variables

The attribute `variables` (no default, required) defines the variables that the command `match` will update while trying to minimize the objective function.

```
variables = { variables-attributes,
              { variable-attributes },
              ...more variable definitions...
              { variable-attributes } },
```

The *variable-attributes* is a set of attributes that specify a single variable:

`var` A *string* specifying the identifier (and indirection) needed to reach the variable from the user's scope where the `match` command is defined. (default: `nil`).
Example: `var = "lhcb1.mq_12l4_b1.k1"`.

`name` A *string* specifying the name of the variable to display when the `info` level is positive. (default: `var`).
Example: `name = "MQ.12L4.B1->k1"`.

`min` A *number* specifying the lower bound for the variable. (default: `-inf`).
Example: `min = -4`.

`max` A *number* specifying the upper bound for the variable. (default: `+inf`).
Example: `max = 10`.

`sign` A *logical* enforcing the sign of the variable by moving `min` or `max` to zero depending on the sign of its initial value. (default: `false`).
Example: `sign = true`.

`slope` A *number* enforcing (LSopt methods only) with its sign the variation direction of the variable, i.e. positive will only increase and negative will only decrease. (default: `0`).
Example: `slope = -1`.

`step` A small positive *number* used to approximate the derivatives using the finite difference method. If the value is not provided, the command will use some heuristic. (default: `nil`).
Example: `step = 1e-6`.

`tol` A *number* specifying the tolerance on the variable step. If an update is smaller than `tol`, the command will return the status `"XTOL"`. (default: `0`).
Example: `tol = 1e-8`.

`get` A *callable* `(e)` returning the variable value as a *number*, optionally using the matching environment passed as first argument. This attribute is required if the variable is *local* or an *upvalue* to avoid a significant slowdown of the code. (default: `nil`).
Example: `get := lhcb1.mq_12l4_b1.k1`.

[2] The function `mchklost` is provided by the GPhys module.

`set`	A *callable* `(v, e)` updating the variable value with the *number* passed as first argument, optionally using the matching environment passed as second argument. This attribute is required if the variable is *local* or an *upvalue* to avoid a significant slowdown of the code. (default: `nil`). Example: `set = \v,e => lhcb1.mqxa_1l5.k1 = v*e.usrdef.xon end`.

The *variables-attributes* is a set of attributes that specify all variables together, but with a lower precedence than the single variable specification of the same name unless otherwise specified:

`min`	Idem *variable-attributes*, but for all variables with no local override.
`max`	Idem *variable-attributes*, but for all variables with no local override.
`sign`	Idem *variable-attributes*, but for all variables with no local override.
`slope`	Idem *variable-attributes*, but for all variables with no local override.
`step`	Idem *variable-attributes*, but for all variables with no local override.
`tol`	Idem *variable-attributes*, but for all variables with no local override.
`rtol`	A *number* specifying the relative tolerance on all variable steps. If an update is smaller than `rtol` relative to its variable value, the command will return the status `"XTOL"`. (default: `eps`). Example: `tol = 1e-8`.
`nvar`	A *number* specifying the number of variables of the problem. It is useful when the problem is made abstract with functions and it is not possible to deduce this count from single variable definitions, or one needs to override it. (default: `nil`). Example: `nvar = 15`.
`get`	A *callable* `(x, e)` updating a *vector* passed as first argument with the values of all variables, optionally using the matching environment passed as second argument. This attribute supersedes all single variable `get` and may be useful when it is better to read all the variables together, or when they are all *local*s or *upvalue*s. (default: `nil`). Example: `get = \x,e -> e.usrdef.var:copy(x)`.
`set`	A *callable* `(x, e)` updating all the variables with the values passed as first argument in a *vector*, optionally using the matching environment passed as second argument. This attribute supersedes all single variable `set` and may be useful when it is better to update all the variables together, or when they are all *local*s or *upvalue*s. (default: `nil`). Example: `set = \x,e -> x:copy(e.usrdef.var)`.
`nowarn`	A *logical* disabling a warning emitted when the definition of `get` and `set` are advised but not defined. It is safe to not define `get` and `set` in such case, but it will significantly slowdown the code. (default: `nil`). Example: `nowarn = true`.

5 Constraints

The attributes `equalities` (default: `{}`) and `inequalities` (default: `{}`) define the constraints that the command `match` will try to satisfy while minimizing the objective function. Equalities and inequalities are considered differently when calculating the penalty function.

```
equalities   = { constraints-attributes,
                 { constraint-attributes },
                 ...more equality definitions...
                 { constraint-attributes } },
inequalities = { constraints-attributes,
                 { constraint-attributes },
                 ...more inequality definitions...
                 { constraint-attributes } },
weights      = { weights-list },
```

The *constraint-attributes* is a set of attributes that specify a single constraint, either an *equality* or an *inequality*:

expr A *callable* `(r, e)` returning the constraint value as a *number*, optionally using the result of `command` passed as first argument, and the matching environment passed as second argument. (default: `nil`)
Example: `expr = \t -> t.IP8.beta11 - beta_ip8`.

name A *string* specifying the name of the constraint to display when the `info` level is positive. (default: `nil`).
Example: `name = "betx@IP8"`.

kind A *string* specifying the kind to refer to for the weight of the constraint, taken either in the user-defined or in the default *weights-list*. (default: `nil`).
Example: `kind = "dq1"`.

weight A *number* used to override the weight of the constraint. (default: `nil`).
Example: `weight = 100`.

tol A *number* specifying the tolerance to apply on the constraint when checking for its fulfillment. (default: `1e-8`).
Example: `tol = 1e-6`.

The *constraints-attributes* is a set of attributes that specify all equalities or inequalities constraints together, but with a lower precedence than the single constraint specification of the same name unless otherwise specified:

tol Idem *constraint-attributes*, but for all constraints with no local override.

nequ A *number* specifying the number of equations (i.e. number of equalities or inequalities) of the problem. It is useful when the problem is made abstract with functions and it is not possible to deduce this count from single constraint definitions, or one needs to override it. (default: `nil`).
Example: `nequ = 15`.

exec A *callable* `(x, c, cjac)` updating a *vector* passed as second argument with the values of all constraints, and updating an optional *matrix* passed as third argument with the Jacobian of all constraints (if not `nil`), using the variables values passed in a *vector* as first argument. This attribute supersedes all constraints `expr` and may be useful when it is better to update all the constraints together. (default: `nil`).
Example: `exec = myinequ`, where (`nvar=2` and `nequ=2`)

```
local function myinequ (x, c, cjac)
   c:fill { 8*x[1]^3 - x[2] ; (1-x[1])^3 - x[2] }
   if cjac then -- fill [2x2] matrix if present
      cjac:fill { 24*x[1]^2, -1 ; -3*(1-x[1])^2, -1 }
   end
end
```

disp A *logical* disabling the display of the equalities in the summary if it is explicitly set to `false`. This is useful for fitting data where equalities are used to compute the residuals. (default: `nil`).
Example: `disp = false`.

The *weights-list* is a set of attributes that specify weights for kinds used by constraints. It allows to override the default weights of the supported kinds summarized in Table 15.2, or to extend this list with new kinds and weights. The default weight for any undefined `kind` is `1`.
Example: `weights = { q1=100, q2=100, mykind=3 }`.

Table 15.2: List of supported kinds (*string*) and their default weights (*number*).

Name	Weight	Name	Weight	Name	Weight	Generic name
x	10	y	10	t	10	
px	100	py	100	pt	100	
dx	10	dy	10	dt	10	d
dpx	100	dpy	100	dpt	100	dp
ddx	10	ddy	10	ddt	10	dd
ddpx	100	ddpy	100	ddpt	100	ddp
wx	1	wy	1	wz	1	w
phix	1	phiy	1	phiz	1	phi
betx	1	bety	1	betz	1	beta
alfx	10	alfy	10	alfz	10	alfa
mux	10	muy	10	muz	10	mu
beta1	1	beta2	1	beta3	1	beta
alfa1	10	alfa2	10	alfa3	10	alfa
mu1	10	mu2	10	mu3	10	mu
q1	10	q2	10	q3	10	q
dq1	1	dq2	1	dq3	1	dq

6 Objective

The attribute `objective` (default: `{}`) defines the objective that the command `match` will try to minimize.

```
objective = { objective-attributes },
```

The *objective-attributes* is a set of attributes that specify the objective to fulfill:

method A *string* specifying the algorithm to use for solving the problem, see Tables 15.3, 15.4 and 15.5. (default: `"LN_COBYLA"` if `objective.exec` is defined, `"LD_JACOBIAN"` otherwise).
Example: `method = "LD_LMDIF"`.

submethod A *string* specifying the algorithm from NLopt module to use for solving the problem locally when the method is an augmented algorithm, see Table 15.4 and 15.5. (default: `"LN_COBYLA"`).
Example: `method = "AUGLAG", submethod = "LD_SLSQP"`.

fmin A *number* corresponding to the minimum to reach during the optimization. For least squares problems, it corresponds to the tolerance on the penalty function. If an iteration find a value smaller than `fmin` and all the constraints are fulfilled, the command will return the status `"FMIN"`. (default: `nil`).
Example: `fmin = 1e-12`.

tol A *number* specifying the tolerance on the objective function step. If an update is smaller than `tol`, the command will return the status `"FTOL"`. (default: `0`).
Example: `tol = 1e-10`.

rtol A *number* specifying the relative tolerance on the objective function step. If an update is smaller than `rtol` relative to its step value, the command will return the status `"FTOL"`. (default: `0`).
Example: `tol = 1e-8`.

bstra A *number* specifying the strategy to select the *best case* of the objective function. (default: `nil`).
Example: `bstra = 0`.[3]

broyden A *logical* allowing the Jacobian approximation by finite difference to update its columns with a *Broyden's rank one* estimates when the step of the corresponding variable is almost collinear with the variables step vector. This option may save some expensive calls to `command`, e.g. save Twiss calculations, when it does not degrade the rate of convergence of the selected method. (default: `nil`).
Example: `broyden = true`.

reset A *logical* specifying to the `match` command to restore the initial state of the variables before returning. This is useful to attempt an optimization without changing the state of the variables. Note that if any function amongst `command`, variables `get` and `set`, constraints `expr` or `exec`, or objective `exec` have side effects on the environment, these will be persistent. (default: `nil`).
Example: `reset = true`.

exec A *callable* `(x, fgrd)` returning the value of the objective function as a *number*, and updating a *vector* passed as second argument with its gradient, using the variables values passed in a *vector* as first argument. (default: `nil`).
Example: `exec = myfun`, where (`nvar=2`)

```
local function myfun(x, fgrd)
   if fgrd then -- fill [2x1] vector if present
      fgrd:fill { 0, 0.5/sqrt(x[2]) }
   end
   return sqrt(x[2])
end
```

grad A *logical* enabling `(true)` or disabling `(false)` the approximation by finite difference of the gradient of the objective function or the Jacobian of the constraints. A `nil` value will

[3]MAD-X matching corresponds to `bstra=0`.

be converted to `true` if no `exec` function is defined and the selected `method` requires derivatives (`D`), otherwise it will be converted to `false`. (default: `nil`).
Example: `grad = false`.

`bisec` A *number* specifying (LSopt methods only) the maximum number of attempt to minimize an increasing objective function by reducing the variables steps by half, i.e. that is a line search using $\alpha = 0.5^k$ where $k = 0..$`bisec`. (default: `3` if `objective.exec` is undefined, `0` otherwise).
Example: `bisec = 9`.

`rcond` A *number* specifying (LSopt methods only) how to determine the effective rank of the Jacobian while solving the least squares system (see `ssolve` from the Matrix module). This attribute can be updated between iterations, e.g. through `env.objective.rcond`. (default: `eps`).
Example: `rcond = 1e-14`.

`jtol` A *number* specifying (LSopt methods only) the tolerance on the norm of the Jacobian rows to reject useless constraints. This attribute can be updated between iterations, e.g. through `env.objective.jtol`. (default: `eps`).
Example: `tol = 1e-14`.

`jiter` A *number* specifying (LSopt methods only) the maximum allowed attempts to solve the least squares system when variables are rejected, e.g. wrong slope or out-of-bound values. (default: `10`).
Example: `jiter = 15`.

`jstra` A *number* specifying (LSopt methods only) the strategy to use for reducing the variables of the least squares system. (default: `1`).
Example: `jstra = 3`.[4]

`jstra`	Strategy for reducing variables of least squares system.
`0`	no variables reduction, constraints reduction is still active.
`1`	reduce system variables for bad slopes and out-of-bound values.
`2`	idem 1, but bad slopes reinitialize variables to their original state.
`3`	idem 2, but strategy switches definitely to 0 if `jiter` is reached.

7 Algorithms

The `match` command supports local and global optimization algorithms through the `method` attribute, as well as combinations of them with the `submethod` attribute (see objective). The method should be selected according to the kind of problem that will add a prefix to the method name: local (`L`) or global (`G`), with (`D`) or without (`N`) derivatives, and least squares or nonlinear function minimization. When the method requires the derivatives (`D`) and no `objective.exec` function is defined or the attribute `grad` is set to `false`, the `match` command will approximate the derivatives, i.e. gradient and Jacobian, by the finite difference method (see derivatives).

Most global optimization algorithms explore the variables domain with methods belonging to stochastic sampling, deterministic scanning, and splitting strategies, or a mix of them. Hence, all global methods require *boundaries* to define the searching region, which may or may not be internally scaled to a hypercube. Some global methods allow to specify with the `submethod` attribute, the local method to use for searching local minima. If this is not the case, it is wise to refine the global solution with a local method

[4]MAD-X `JACOBIAN` with `strategy=3` corresponds to `jstra=3`.

afterward, as global methods put more effort on finding global solutions than precise local minima. The global (`G`) optimization algorithms, with (`D`) or without (`N`) derivatives, are listed in Table 15.5.

Most local optimization algorithms with derivatives are variants of the Newton iterative method suitable for finding local minima of nonlinear vector-valued function $\mathbf{f}(\mathbf{x})$, i.e. searching for stationary points. The iteration steps $\mathbf{h}$ are given by the minimization $\mathbf{h} = -\alpha(\nabla^2\mathbf{f})^{-1}\nabla\mathbf{f}$, coming from the local approximation of the function at the point $\mathbf{x}+\mathbf{h}$ by its Taylor series truncated at second order $\mathbf{f}(\mathbf{x}+\mathbf{h}) \approx \mathbf{f}(\mathbf{x}) + \mathbf{h}^T\nabla\mathbf{f}(\mathbf{x}) + \frac{1}{2}\mathbf{h}^T\nabla^2\mathbf{f}(\mathbf{x})\mathbf{h}$, and solved for $\nabla_{\mathbf{h}}\mathbf{f} = 0$. The factor $\alpha > 0$ is part of the line search strategy, which is sometimes replaced or combined with a trusted region strategy like in the Leverberg-Marquardt algorithm. The local (`L`) optimization algorithms, with (`D`) or without (`N`) derivatives, are listed in Table 15.3 for least squares methods and in Table 15.4 for non-linear methods, and can be grouped by family of algorithms:

`Newton` An iterative method to solve nonlinear systems that uses iteration step given by the minimization $\mathbf{h} = -\alpha(\nabla^2\mathbf{f})^{-1}\nabla\mathbf{f}$.

`Newton-Raphson` An iterative method to solve nonlinear systems that uses iteration step given by the minimization $\mathbf{h} = -\alpha(\nabla\mathbf{f})^{-1}\mathbf{f}$.

`Gradient-Descent` An iterative method to solve nonlinear systems that uses iteration step given by $\mathbf{h} = -\alpha\nabla\mathbf{f}$.

`Quasi-Newton` A variant of the Newton method that uses BFGS approximation of the Hessian $\nabla^2\mathbf{f}$ or its inverse $(\nabla^2\mathbf{f})^{-1}$, based on values from past iterations.

`Gauss-Newton` A variant of the Newton method for *least-squares* problems that uses iteration step given by the minimization $\mathbf{h} = -\alpha(\nabla\mathbf{f}^T\nabla\mathbf{f})^{-1}(\nabla\mathbf{f}^T\mathbf{f})$, where the Hessian $\nabla^2\mathbf{f}$ is approximated by $\nabla\mathbf{f}^T\nabla\mathbf{f}$ with $\nabla\mathbf{f}$ being the Jacobian of the residuals $\mathbf{f}$.

`Levenberg-Marquardt` A hybrid G-N and G-D method for *least-squares* problems that uses iteration step given by the minimization $\mathbf{h} = -\alpha(\nabla\mathbf{f}^T\nabla\mathbf{f} + \mu\mathbf{D})^{-1}(\nabla\mathbf{f}^T\mathbf{f})$, where $\mu > 0$ is the damping term selecting the method G-N (small μ) or G-D (large μ), and $\mathbf{D} = \mathrm{diag}(\nabla\mathbf{f}^T\nabla\mathbf{f})$.

`Simplex` A linear programming method (simplex method) working without using any derivatives.

`Nelder-Mead` A nonlinear programming method (downhill simplex method) working without using any derivatives.

`Principal-Axis` An adaptive coordinate descent method working without using any derivatives, selecting the descent direction from the Principal Component Analysis.

7.1 Stopping criteria

The `match` command will stop the iteration of the algorithm and return one of the following `status` if the corresponding criteria, *checked in this order*, is fulfilled (see also Table 15.1):

`STOPPED` Check `env.stop == true`, i.e. termination forced by a user-defined function.

`FMIN` Check $f \leqslant f_{\text{min}}$ if $c_{\text{fail}} = 0$ or `bstra == 0`, where f is the current value of the objective function, and c_{fail} the number of failed constraints (i.e. feasible point).

`FTOL` Check $|\Delta f| \leqslant f_{\text{tol}}$ or $|\Delta f| \leqslant f_{\text{rtol}}\,|f|$ if $c_{\text{fail}} = 0$, where f and Δf are the current value and step of the objective function, and c_{fail} the number of failed constraints (i.e. feasible point).

`XTOL`	Check $\max(\lvert\Delta\mathbf{x}\rvert - \mathbf{x}_{\text{tol}}) \leqslant 0$ or $\max(\lvert\Delta\mathbf{x}\rvert - \mathbf{x}_{\text{rtol}} \circ \lvert\mathbf{x}\rvert) \leqslant 0$, where $\mathbf{x}$ and $\Delta\mathbf{x}$ are the current values and steps of the variables. Note that these criteria are checked even for non feasible points, i.e. $c_{\text{fail}} > 0$, as the algorithm can be trapped in a local minima that does not satisfy the constraints.
`ROUNDOFF`	Check $\max(\lvert\Delta\mathbf{x}\rvert - \varepsilon\,\lvert\mathbf{x}\rvert) \leqslant 0$ if $\mathbf{x}_{\text{rtol}} < \varepsilon$, where $\mathbf{x}$ and $\Delta\mathbf{x}$ are the current values and steps of the variables. The LSopt module returns also this status if the Jacobian is full of zeros, which is `jtol` dependent during its `jstra` reductions.
`MAXCALL`	Check `env.ncall >= maxcall if maxcall > 0`.
`MAXTIME`	Check `env.dtime >= maxtime if maxtime > 0`.

7.2 Objective function

The objective function is the key point of the `match` command, specially when tolerances are applied to it or to the constraints, or the best case strategy is changed. It is evaluated as follow:

1. Update user's `variables` with the *vector* $\mathbf{x}$.
2. Evaluate the *callable* `command` if defined and pass its value to the constraints.
3. Evaluate the *callable* `objective.exec` if defined and save its value f.
4. Evaluate the *callable* `equalities.exec` if defined, otherwise evaluate all the functions `equalities[].expr(cmd` and use the result to fill the *vector* $\mathbf{c}^{=}$.
5. Evaluate the *callable* `inequalities.exec` if defined, otherwise evaluate all the functions `inequalities[].expr` and use the result to fill the *vector* $\mathbf{c}^{\leqslant}$.
6. Count the number of invalid constraints $c_{\text{fail}} = \text{card}\{\lvert\mathbf{c}^{=}\rvert > \mathbf{c}^{=}_{\text{tol}}\} + \text{card}\{\mathbf{c}^{\leqslant} > \mathbf{c}^{\leqslant}_{\text{tol}}\}$.[5]
7. Calculate the *penalty* $p = \lVert\mathbf{c}\rVert / \lVert\mathbf{w}\rVert$, where $\mathbf{c} = \mathbf{w} \circ \begin{bmatrix}\mathbf{c}^{=}\\ \mathbf{c}^{\leqslant}\end{bmatrix}$ and $\mathbf{w}$ is the weights *vector* of the constraints. Set $f = p$ if the *callable* `objective.exec` is undefined.[6]
8. Save the current iteration state as the best state depending on the strategy `bstra`. The default `bstra=nil` corresponds to the last strategy.

`bstra`	Strategy for selecting the best case of the objective function.
`0`	$f < f^{\text{best}}_{\text{min}}$, no feasible point check.
`1`	$c_{\text{fail}} \leqslant c^{\text{best}}_{\text{fail}}$ and $f < f^{\text{best}}_{\text{min}}$, improve both feasible point and objective.
`-`	$c_{\text{fail}} < c^{\text{best}}_{\text{fail}}$ or $c_{\text{fail}} = c^{\text{best}}_{\text{fail}}$ and $f < f^{\text{best}}_{\text{min}}$, improve feasible point or objective.

7.3 Derivatives

The derivatives are approximated by the finite difference methods when the selected algorithm requires them (`D`) and the function `objective.exec` is undefined or the attribute `grad=false`. The difficulty of the finite difference methods is to choose the small step h for the difference. The `match` command uses the *forward difference method* with a step $h = 10^{-4}\,\lVert\mathbf{h}\rVert$, where $\mathbf{h}$ is the last iteration steps, unless it is overridden by the user with the variable attribute `step`. In order to avoid zero step size, which would be problematic for the calculation of the Jacobian, the choice of h is a bit more subtle:

$$\frac{\partial f_j}{\partial x_i} \approx \frac{f_j(\mathbf{x} + h\mathbf{e_i}) - f_j(\mathbf{x})}{h} \quad ; \quad h = \begin{cases} 10^{-4}\,\lVert\mathbf{h}\rVert & \text{if } \lVert\mathbf{h}\rVert \neq 0 \\ 10^{-8}\,\lVert\mathbf{x}\rVert & \text{if } \lVert\mathbf{h}\rVert = 0 \text{ and } \lVert\mathbf{x}\rVert \neq 0 \\ 10^{-10} & \text{otherwise.} \end{cases}$$

[5] The LSopt module sets the values of valid inequalities to zero, i.e. $\mathbf{c}^{\leqslant} = 0$ if $\mathbf{c}^{\leqslant} \leqslant \mathbf{c}^{\leqslant}_{\text{tol}}$.

[6] The penalty is the norm of the residuals $\lVert\mathbf{c}\rVert$, not the usual $\frac{1}{2}\,\lVert\mathbf{c}\rVert^2$, which affects tolerance specification.

Hence the approximation of the Jacobian will need an extra evaluation of the objective function per variable. If this evaluation has an heavy cost, e.g. like a `twiss` command, it is possible to approximate the Jacobian evolution by a Broyden's rank-1 update with the `broyden` attribute:

$$\mathbf{J}_{k+1} = \mathbf{J}_k + \frac{\mathbf{f}(\mathbf{x}_k + \mathbf{h}_k) - \mathbf{f}(\mathbf{x}_k) - \mathbf{J}_k\,\mathbf{h}_k}{\|\mathbf{h}_k\|^2}\,\mathbf{h}_k^T$$

The update of the i-th column of the Jacobian by the Broyden approximation makes sense if the angle between $\mathbf{h}$ and $\mathbf{e}_i$ is small, that is when $|\mathbf{h}^T\mathbf{e}_i| \geqslant \gamma\,\|\mathbf{h}\|$. The `match` command uses a rather pessimistic choice of $\gamma = 0.8$, which gives good performance. Nevertheless, it is advised to always check if Broyden's update saves evaluations of the objective function for your study.

8 Console output

The verbosity of the output of the `match` command on the console (e.g. terminal) is controlled by the `info` level, where the level `info=0` means a completely silent command as usual. The first verbose level `info=1` displays the *final summary* at the end of the matching, as shown in the Figure 15.2, and the next level `info=2` adds *intermediate summary* for each evaluation of the objective function, as shown in the Figure 15.3. The columns of these tables are self-explanatory, and the sign `>` on the right of the constraints marks those failing.

The bottom line of the *intermediate summary* displays in order:

– the number of evaluation of the objective function so far,
– the elapsed time in second (in square brackets) so far,
– the current objective function value,
– the current objective function step,
– the current number of constraint that failed c_{fail}.

The bottom line of the *final summary* displays the same information but for the best case found, as well as the final status returned by the `match` command. The number in square brackets right after `fbst` is the evaluation number of the best case.

The LSopt module adds the sign `#` to mark the *adjusted* variables and the sign $*$ to mark the *rejected* variables and constraints on the right of the *intermediate summary* tables to qualify the behavior of the constraints and the variables during the optimization process. If these signs appear in the *final summary* too, it means that they were always adjusted or rejected during the matching, which is useful to tune your study e.g. by removing the useless constraints.

9 Modules

The `match` command can be extended easily with new optimizer either from external libraries or internal module, or both. The interface should be flexible and extensible enough to support new algorithms and new options with a minimal effort.

9.1 LSopt

The LSopt (Least Squares optimization) module implements custom variant of the Newton-Raphson and the Levenberg-Marquardt algorithms to solve least squares problems. Both support the options `rcond`, `bisec`, `jtol`, `jiter` and `jstra` described in the section objective, with the same default values. Table 15.3 lists the names of the algorithms for the attribute `method`. These algorithms cannot be used with the attribute `submethod` for the augmented algorithms of the NLopt module, which would not make sense as these methods support both equalities and inequalities.

Figure 15.2: Match command summary output (info=1).

```
 Constraints                Type        Kind        Weight     Penalty Value
-----------------------------------------------------------------------------
 1 IP8                      equality    beta        1          9.41469e-14
 2 IP8                      equality    beta        1          3.19744e-14
 3 IP8                      equality    alfa        10         0.00000e+00
 4 IP8                      equality    alfa        10         1.22125e-14
 5 IP8                      equality    dx          10         5.91628e-14
 6 IP8                      equality    dpx         100        1.26076e-13
 7 E.DS.R8.B1               equality    beta        1          7.41881e-10
 8 E.DS.R8.B1               equality    beta        1          1.00158e-09
 9 E.DS.R8.B1               equality    alfa        10         4.40514e-12
10 E.DS.R8.B1               equality    alfa        10         2.23532e-11
11 E.DS.R8.B1               equality    dx          10         7.08333e-12
12 E.DS.R8.B1               equality    dpx         100        2.12877e-13
13 E.DS.R8.B1               equality    mu1         10         2.09610e-12
14 E.DS.R8.B1               equality    mu2         10         1.71063e-12

 Variables                  Final Value  Init. Value  Lower Limit  Upper Limit
--------------------------------------------------------------------------------
 1 kq4.l8b1                -3.35728e-03 -4.31524e-03 -8.56571e-03  0.00000e+00
 2 kq5.l8b1                 4.93618e-03  5.28621e-03  0.00000e+00  8.56571e-03
 3 kq6.l8b1                -5.10313e-03 -5.10286e-03 -8.56571e-03  0.00000e+00
 4 kq7.l8b1                 8.05555e-03  8.25168e-03  0.00000e+00  8.56571e-03
 5 kq8.l8b1                -7.51668e-03 -5.85528e-03 -8.56571e-03  0.00000e+00
 6 kq9.l8b1                 7.44662e-03  7.07113e-03  0.00000e+00  8.56571e-03
 7 kq10.l8b1               -6.73001e-03 -6.39311e-03 -8.56571e-03  0.00000e+00
 8 kqtl11.l8b1              6.85635e-04  7.07398e-04  0.00000e+00  5.56771e-03
 9 kqt12.l8b1              -2.38722e-03 -3.08650e-03 -5.56771e-03  0.00000e+00
10 kqt13.l8b1               5.55969e-03  3.78543e-03  0.00000e+00  5.56771e-03
11 kq4.r8b1                 4.23719e-03  4.39728e-03  0.00000e+00  8.56571e-03
12 kq5.r8b1                -5.02348e-03 -4.21383e-03 -8.56571e-03  0.00000e+00
13 kq6.r8b1                 4.18341e-03  4.05914e-03  0.00000e+00  8.56571e-03
14 kq7.r8b1                -5.48774e-03 -6.65981e-03 -8.56571e-03  0.00000e+00
15 kq8.r8b1                 5.88978e-03  6.92571e-03  0.00000e+00  8.56571e-03
16 kq9.r8b1                -3.95756e-03 -7.46154e-03 -8.56571e-03  0.00000e+00
17 kq10.r8b1                7.18012e-03  7.55573e-03  0.00000e+00  8.56571e-03
18 kqtl11.r8b1             -3.99902e-03 -4.78966e-03 -5.56771e-03  0.00000e+00
19 kqt12.r8b1              -1.95221e-05 -1.74210e-03 -5.56771e-03  0.00000e+00
20 kqt13.r8b1              -2.04425e-03 -3.61438e-03 -5.56771e-03  0.00000e+00

ncall=381 [4.1s], fbst[381]=8.80207e-12, fstp=-3.13047e-08, status=FMIN.
```

9.2 NLopt

The NLopt (Non-Linear optimization) module provides a simple interface to the algorithms implemented in the embedded NLopt library. Tables 15.4 and 15.5 list the names of the local and global algorithms

Figure 15.3: Match command intermediate output (info=2).

```
 Constraints                Type        Kind        Weight     Penalty Value
-----------------------------------------------------------------------------
 1 IP8                      equality    beta        1          3.10118e+00 >
 2 IP8                      equality    beta        1          1.85265e+00 >
 3 IP8                      equality    alfa        10         9.77591e-01 >
 4 IP8                      equality    alfa        10         8.71014e-01 >
 5 IP8                      equality    dx          10         4.37803e-02 >
 6 IP8                      equality    dpx         100        4.59590e-03 >
 7 E.DS.R8.B1               equality    beta        1          9.32093e+01 >
 8 E.DS.R8.B1               equality    beta        1          7.60213e+01 >
 9 E.DS.R8.B1               equality    alfa        10         2.98722e+00 >
10 E.DS.R8.B1               equality    alfa        10         1.04758e+00 >
11 E.DS.R8.B1               equality    dx          10         7.37813e-02 >
12 E.DS.R8.B1               equality    dpx         100        6.67388e-03 >
13 E.DS.R8.B1               equality    mu1         10         7.91579e-02 >
14 E.DS.R8.B1               equality    mu2         10         6.61916e-02 >

 Variables                  Curr. Value  Curr. Step   Lower Limit  Upper Limit
--------------------------------------------------------------------------------
 1 kq4.l8b1                -3.36997e-03 -4.81424e-04 -8.56571e-03  0.00000e+00 #
 2 kq5.l8b1                 4.44028e-03  5.87400e-04  0.00000e+00  8.56571e-03
 3 kq6.l8b1                -4.60121e-03 -6.57316e-04 -8.56571e-03  0.00000e+00 #
 4 kq7.l8b1                 7.42273e-03  7.88826e-04  0.00000e+00  8.56571e-03
 5 kq8.l8b1                -7.39347e-03  0.00000e+00 -8.56571e-03  0.00000e+00 *
 6 kq9.l8b1                 7.09770e-03  2.58912e-04  0.00000e+00  8.56571e-03
 7 kq10.l8b1               -5.96101e-03 -8.51573e-04 -8.56571e-03  0.00000e+00 #
 8 kqtl11.l8b1              6.15659e-04  8.79512e-05  0.00000e+00  5.56771e-03 #
 9 kqt12.l8b1              -2.66538e-03  0.00000e+00 -5.56771e-03  0.00000e+00 *
10 kqt13.l8b1               4.68776e-03  0.00000e+00  0.00000e+00  5.56771e-03 *
11 kq4.r8b1                 4.67515e-03 -5.55795e-04  0.00000e+00  8.56571e-03 #
12 kq5.r8b1                -4.71987e-03  5.49407e-04 -8.56571e-03  0.00000e+00 #
13 kq6.r8b1                 4.68747e-03 -5.54035e-04  0.00000e+00  8.56571e-03 #
14 kq7.r8b1                -5.35315e-03  4.58938e-04 -8.56571e-03  0.00000e+00 #
15 kq8.r8b1                 5.77068e-03  0.00000e+00  0.00000e+00  8.56571e-03 *
16 kq9.r8b1                -4.97761e-03 -7.11087e-04 -8.56571e-03  0.00000e+00 #
17 kq10.r8b1                6.90543e-03  4.33052e-04  0.00000e+00  8.56571e-03
18 kqtl11.r8b1             -4.16758e-03 -5.95369e-04 -5.56771e-03  0.00000e+00 #
19 kqt12.r8b1              -1.57183e-03  0.00000e+00 -5.56771e-03  0.00000e+00 *
20 kqt13.r8b1              -2.57565e-03  0.00000e+00 -5.56771e-03  0.00000e+00 *

ncall=211 [2.3s], fval=8.67502e-01, fstp=-2.79653e+00, ccnt=14.
```

respectively for the attribute `method`. The methods that do not support equalities (column Equ) or inequalities (column Iqu) can still be used with constraints by specifying them as the `submethod` of the AUGmented LAGrangian `method`. For details about these algorithms, please refer to the Algorithms section of its online documentation.

Table 15.3: List of supported least squares methods (LSopt).

method	Equ	Iqu	Description
LD_JACOBIAN	y	y	Modified Newton-Raphson algorithm.
LD_LMDIF	y	y	Modified Levenberg-Marquardt algorithm.

Table 15.4: List of supported non-linear local methods (NLopt).

method	Equ	Iqu	Description
			Local optimizers without derivative (LN_)
LN_BOBYQA	n	n	Bound-constrained Optimization BY Quadratic Approximations algorithm.
LN_COBYLA	y	y	Bound Constrained Optimization BY Linear Approximations algorithm.
LN_NELDERMEAD	n	n	Original Nelder-Mead algorithm.
LN_NEWUOA	n	n	Older and less efficient LN_BOBYQA.
LN_NEWUOA_BOUND	n	n	Older and less efficient LN_BOBYQA with bound constraints.
LN_PRAXIS	n	n	PRincipal-AXIS algorithm.
LN_SBPLX	n	n	Subplex algorithm, variant of Nelder-Mead.
			Local optimizers with derivative (LD_)
LD_CCSAQ	n	y	Conservative Convex Separable Approximation with Quatratic penalty.
LD_LBFGS	n	n	BFGS algorithm with low memory footprint.
LD_LBFGS_NOCEDAL	n	n	Variant from J. Nocedal of LD_LBFGS.
LD_MMA	n	y	Method of Moving Asymptotes algorithm.
LD_SLSQP	y	y	Sequential Least-Squares Quadratic Programming algorithm.
LD_TNEWTON	n	n	Inexact Truncated Newton algorithm.
LD_TNEWTON_PRECOND	n	n	Idem LD_TNEWTON with preconditioning.
LD_TNEWTON_PRECOND-_RESTART	n	n	Idem LD_TNEWTON with preconditioning and steepest-descent restarting.
LD_TNEWTON_RESTART	n	n	Idem LD_TNEWTON with steepest-descent restarting.
LD_VAR1	n	n	Shifted limited-memory VARiable-metric rank-1 algorithm.
LD_VAR2	n	n	Shifted limited-memory VARiable-metric rank-2 algorithm.

10 Examples

10.1 Matching tunes and chromaticity

The following example below shows how to match the betatron tunes of the LHC beam 1 to $q_1 = 64.295$ and $q_2 = 59.301$ using the quadrupoles strengths kqtf and kqtd, followed by the matching of the chromaticities to $dq_1 = 15$ and $dq_2 = 15$ using the main sextupole strengths ksf and ksd.

```
local lhcb1 in MADX
local twiss, match in MAD

local status, fmin, ncall = match {
  command   := twiss { sequence=lhcb1, cofind=true,
                       method=4, observe=1 },
  variables  = { rtol=1e-6, -- 1 ppm
```

Table 15.5: List of supported non-linear global methods (NLopt).

method	Equ	Iqu	Description
Global optimizers without derivative (GN_)			
GN_CRS2_LM	n	n	Variant of the Controlled Random Search algorithm with Local Mutation (mixed stochastic and genetic method).
GN_DIRECT	n	n	DIviding RECTangles algorithm (deterministic method).
GN_DIRECT_L	n	n	Idem GN_DIRECT with locally biased optimization.
GN_DIRECT_L_RAND	n	n	Idem GN_DIRECT_L with some randomization in the selection of the dimension to reduce next.
GN_DIRECT*_NOSCAL	n	n	Variants of above GN_DIRECT* without scaling the problem to a unit hypercube to preserve dimension weights.
GN_ESCH	n	n	Modified Evolutionary algorithm (genetic method).
GN_ISRES	y	y	Improved Stochastic Ranking Evolution Strategy algorithm (mixed genetic and variational method).
GN_MLSL	n	n	Multi-Level Single-Linkage algorithm (stochastic method).
GN_MLSL_LDS	n	n	Idem GN_MLSL with low-discrepancy scan sequence.
Global optimizers with derivative (GD_)			
GD_MLSL	n	n	Multi-Level Single-Linkage algorithm (stochastic method).
GD_MLSL_LDS	n	n	Idem GL_MLSL with low-discrepancy scan sequence.
GD_STOGO	n	n	Branch-and-bound algorithm (deterministic method).
GD_STOGO_RAND	n	n	Variant of GD_STOGO (deterministic and stochastic method).
Augmented methods for other NLopt optimizers			
AUGLAG	y	y	Augmented Lagrangian algorithm, combines objective function and nonlinear constraints into a single "penalty" function.
AUGLAG_EQ	y	n	Idem AUGLAG but handles only equality constraints and pass inequality constraints to submethod.
G_MLSL	n	n	MLSL with user-specified local algorithm using submethod.
G_MLSL_LDS	n	n	Idem G_MLSL with low-discrepancy scan sequence.

```
                 { var='MADX.kqtf_b1' },
                 { var='MADX.kqtd_b1' }},
  equalities = {{ expr=\t -> t.q1-64.295, name='q1' },
                 { expr=\t -> t.q2-59.301, name='q2' }},
  objective  = { fmin=1e-10, broyden=true },
  maxcall=100, info=2
}
local status, fmin, ncall = match {
  command   := twiss { sequence=lhcb1, cofind=true, chrom=true,
                       method=4, observe=1 },
  variables  = { rtol=1e-6, -- 1 ppm
                 { var='MADX.ksf_b1' },
                 { var='MADX.ksd_b1' }},
  equalities = {{ expr=\t -> t.dq1-15, name='dq1' },
                 { expr=\t -> t.dq2-15, name='dq2' }},
  objective  = { fmin=1e-8, broyden=true },
  maxcall=100, info=2
}
```

10.2 Matching interaction point

The following example hereafter shows how to squeeze the beam 1 of the LHC to $\beta^* = \texttt{beta\_ip8} * 0.6^2$ at the IP8 while enforcing the required constraints at the interaction point and the final dispersion suppressor (i.e. at makers "IP8" and "E.DS.R8.B1") in two iterations, using the 20 quadrupoles strengths from kq4 to kqt13 on left and right sides of the IP. The boundary conditions are specified by the beta0 blocks bir8b1 for the initial conditions and eir8b1 for the final conditions. The final summary and an instance of the intermediate summary of this match example are shown in the Figures 15.2 and 15.3.

```
local SS, ES = "S.DS.L8.B1", "E.DS.R8.B1"
lhcb1.range = SS.."/"..ES
for n=1,2 do
  beta_ip8 = beta_ip8*0.6
  local status, fmin, ncall = match {
    command := twiss { sequence=lhcb1, X0=bir8b1, method=4, observe=1 },
    variables = { sign=true, rtol=1e-8, -- 20 variables
      { var='MADX.kq4_l8b1', name='kq4.l8b1', min=-lim2, max=lim2 },
      { var='MADX.kq5_l8b1', name='kq5.l8b1', min=-lim2, max=lim2 },
      { var='MADX.kq6_l8b1', name='kq6.l8b1', min=-lim2, max=lim2 },
      { var='MADX.kq7_l8b1', name='kq7.l8b1', min=-lim2, max=lim2 },
      { var='MADX.kq8_l8b1', name='kq8.l8b1', min=-lim2, max=lim2 },
      { var='MADX.kq9_l8b1', name='kq9.l8b1', min=-lim2, max=lim2 },
      { var='MADX.kq10_l8b1', name='kq10.l8b1', min=-lim2, max=lim2 },
      { var='MADX.kqtl11_l8b1', name='kqtl11.l8b1', min=-lim3, max=lim3 },
      { var='MADX.kqt12_l8b1', name='kqt12.l8b1' , min=-lim3, max=lim3 },
      { var='MADX.kqt13_l8b1', name='kqt13.l8b1', min=-lim3, max=lim3 },
      { var='MADX.kq4_r8b1', name='kq4.r8b1', min=-lim2, max=lim2 },
      { var='MADX.kq5_r8b1', name='kq5.r8b1', min=-lim2, max=lim2 },
      { var='MADX.kq6_r8b1', name='kq6.r8b1', min=-lim2, max=lim2 },
      { var='MADX.kq7_r8b1', name='kq7.r8b1', min=-lim2, max=lim2 },
      { var='MADX.kq8_r8b1', name='kq8.r8b1', min=-lim2, max=lim2 },
      { var='MADX.kq9_r8b1', name='kq9.r8b1', min=-lim2, max=lim2 },
      { var='MADX.kq10_r8b1', name='kq10.r8b1', min=-lim2, max=lim2 },
      { var='MADX.kqtl11_r8b1', name='kqtl11.r8b1', min=-lim3, max=lim3 },
      { var='MADX.kqt12_r8b1', name='kqt12.r8b1', min=-lim3, max=lim3 },
      { var='MADX.kqt13_r8b1', name='kqt13.r8b1', min=-lim3, max=lim3 },
    },
    equalities = { -- 14 equalities
      { expr=\t -> t.IP8.beta11-beta_ip8, kind='beta', name='IP8' },
      { expr=\t -> t.IP8.beta22-beta_ip8, kind='beta', name='IP8' },
      { expr=\t -> t.IP8.alfa11, kind='alfa', name='IP8' },
      { expr=\t -> t.IP8.alfa22, kind='alfa', name='IP8' },
      { expr=\t -> t.IP8.dx, kind='dx', name='IP8' },
      { expr=\t -> t.IP8.dpx, kind='dpx', name='IP8' },
      { expr=\t -> t[ES].beta11-eir8b1.beta11, kind='beta', name=ES },
      { expr=\t -> t[ES].beta22-eir8b1.beta22, kind='beta', name=ES },
      { expr=\t -> t[ES].alfa11-eir8b1.alfa11, kind='alfa', name=ES },
      { expr=\t -> t[ES].alfa22-eir8b1.alfa22, kind='alfa', name=ES },
      { expr=\t -> t[ES].dx-eir8b1.dx, kind='dx', name=ES },
      { expr=\t -> t[ES].dpx-eir8b1.dpx, kind='dpx', name=ES },
      { expr=\t -> t[ES].mu1-muxip8, kind='mu1', name=ES },
      { expr=\t -> t[ES].mu2-muyip8, kind='mu2', name=ES },
    },
    objective = { fmin=1e-10, broyden=true },
    maxcall=1000, info=2
  }
  MADX.n, MADX.tar = n, fmin
end
```

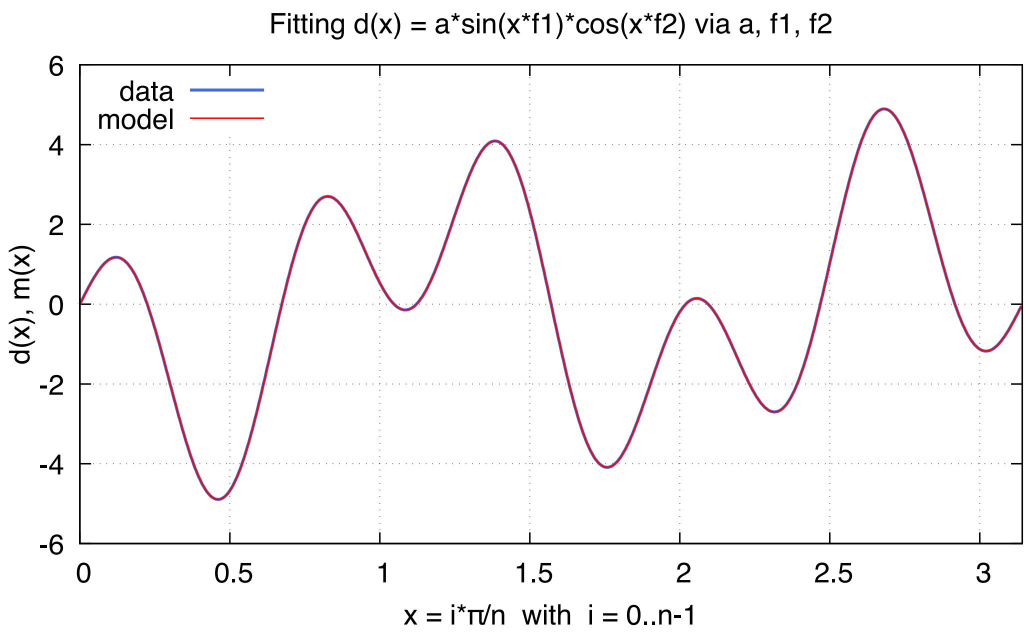
10.3 Fitting data

The following example shows how to fit data with a non-linear model using the least squares methods. The "measurements" are generated by the data function:

$$d(x) = a\sin(xf_1)\cos(xf_2), \quad \text{with} \quad a = 5, f1 = 3, f2 = 7, \text{ and } x \in [0, \pi).$$

The least squares minimization is performed by the small code below starting from the arbitrary values $a = 1$, $f_1 = 1$, and $f_2 = 1$. The `'LD_JACOBIAN'` methods finds the values $a = 5 \pm 10^{-10}$, $f_1 = 3 \pm 10^{-11}$, and $f_2 = 7 \pm 10^{-11}$ in 2574 iterations and 0.1 s. The `'LD_LMDIF'` method finds similar values in 2539 iterations. The data and the model are plotted in the Figure 15.4.

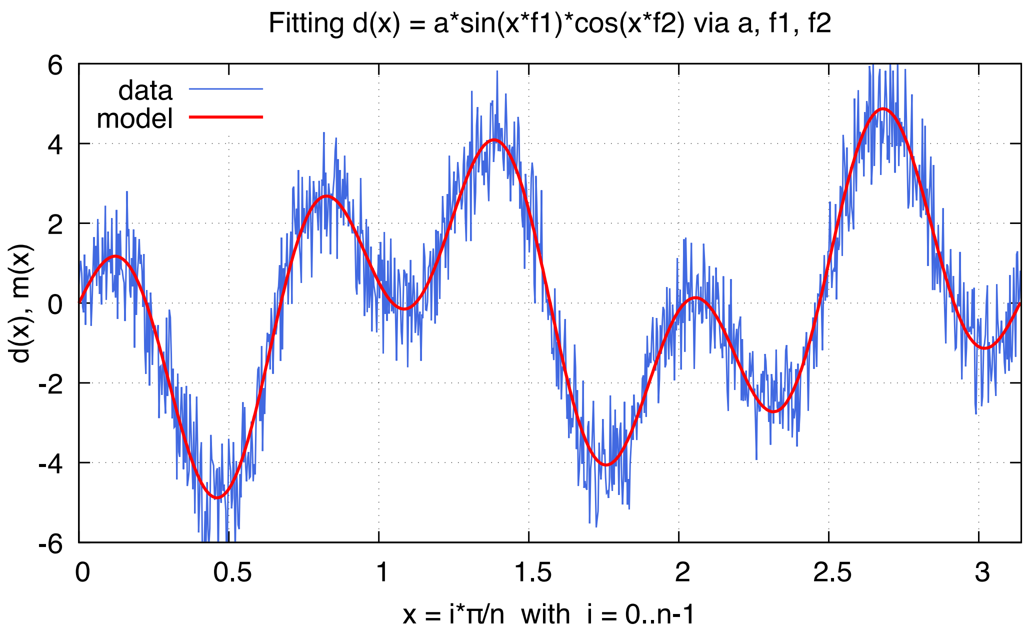
Figure 15.4: Fitting data using the Jacobian or Levenberg-Marquardt methods.



```
local n, k, a, f1, f2 = 1000, pi/1000, 5, 3, 7
local d = vector(n):seq():map \i -> a*sin(i*k*f1)*cos(i*k*f2) -- data
if noise then d=d:map \x -> x+randtn(noise) end -- add noise if any
local m, p = vector(n), { a=1, f1=1, f2=1 } -- model parameters
local status, fmin, ncall = match {
  command   := m:seq():map \i -> p.a*sin(i*k*p.f1)*cos(i*k*p.f2),
  variables  = { { var='p.a'  },
                 { var='p.f1' },
                 { var='p.f2' }, min=1, max=10 },
  equalities = { { expr=\m -> ((d-m):norm()) } },
  objective  = { fmin=1e-9, bisec=noise and 5 },
  maxcall=3000, info=1
}
```

The same least squares minimization can be achieved on noisy data by adding a gaussian RNG truncated at 2σ to the data generator, i.e. `noise=2`, and by increasing the attribute `bisec=5`. Of course, the penalty tolerance `fmin` must be moved to variables tolerance `tol` or `rtol`. The `'LD_JACOBIAN'` methods finds the values $a = 4.98470$, $f_1 = 3.00369$, and $f_2 = 6.99932$ in 704 iterations (404 for `'LD_LMDIF'`). The data and the model are plotted in the Figure 15.5.

Figure 15.5: Fitting data with noise using Jacobian or Levenberg-Marquardt methods.

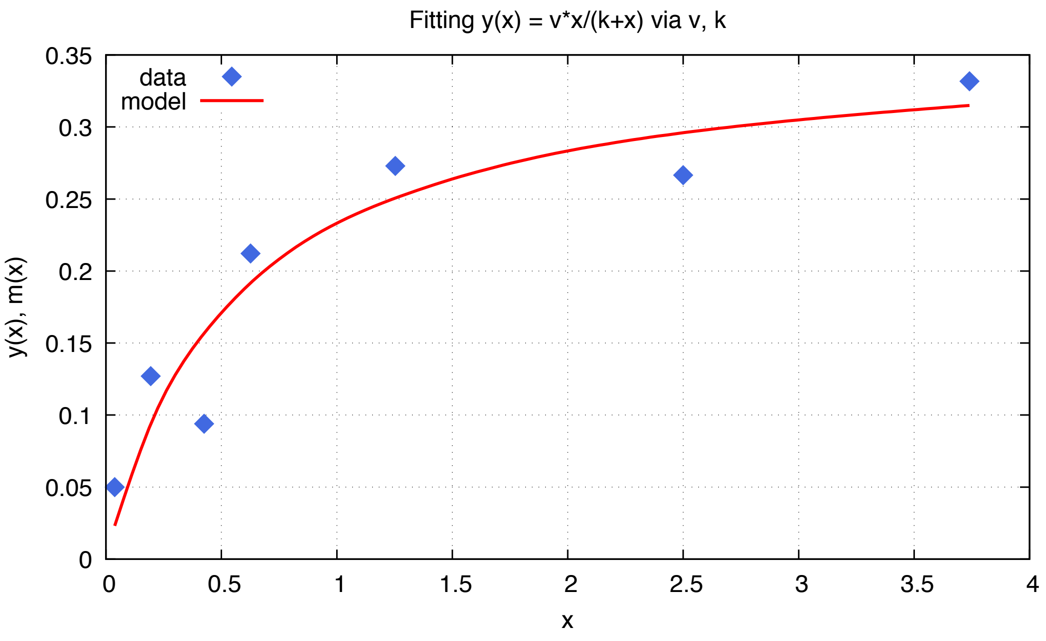


10.4 Fitting data with derivatives

The following example shows how to fit data with a non-linear model and its derivatives using the least squares methods. The least squares minimization is performed by the small code below starting from the arbitrary values $v = 0.9$ and $k = 0.2$. The `'LD_JACOBIAN'` methods finds the values $v = 0.362 \pm 10^{-3}$ and $k = 0.556 \pm 10^{-3}$ in 6 iterations. The `'LD_LMDIF'` method finds similar values in 6 iterations too. The data (points) and the model (curve) are plotted in the Figure 15.6, where the latter has been smoothed using cubic splines.

```
local x = vector{0.038, 0.194, 0.425, 0.626 , 1.253 , 2.500 , 3.740 }
local y = vector{0.050, 0.127, 0.094, 0.2122, 0.2729, 0.2665, 0.3317}
local p = { v=0.9, k=0.2 }
local n = #x
local function eqfun (_, r, jac)
  local v, k in p
  for i=1,n do
    r[i] = y[i] - v*x[i]/(k+x[i])
    jac[2*i-1] = -x[i]/(k+x[i])
    jac[2*i] = v*x[i]/(k+x[i])^2
```

Figure 15.6: Fitting data with derivatives using the Jacobian or Levenberg-Marquardt methods.



```
  end
end
local status, fmin, ncall = match {
  variables  = { tol=5e-3, min=0.1, max=2,
                 { var='p.v' },
                 { var='p.k' } },
  equalities = { nequ=n, exec=eqfun, disp=false },
  maxcall=20
}
```

10.5 Minimizing function

The following example[7] hereafter shows how to find the minimum of the function:

$$\min_{\mathbf{x}\in\mathbb{R}^2} \sqrt{x_2}, \quad \text{subject to the constraints} \quad \begin{cases} x_2 \geqslant 0, \\ x_2 \geqslant (a_1 x_1 + b_1)^3, \\ x_2 \geqslant (a_2 x_1 + b_2)^3, \end{cases}$$

for the parameters $a_1 = 2, b_1 = 0, a_2 = -1$ and $b_2 = 1$. The minimum of the function is $f_{\min} = \sqrt{\frac{8}{27}}$ at the point $\mathbf{x} = (\frac{1}{3}, \frac{8}{27})$, and found by the method LD_MMA in 11 evaluations for a relative tolerance of 10^{-4} on the variables, starting at the arbitrary point $\mathbf{x}_0 = (1.234, 5.678)$.

```
local function testFuncFn (x, grd)
  if grd then x:fill{ 0, 0.5/sqrt(x[2]) } end
  return sqrt(x[2])
end
local function testFuncLe (x, r, jac)
  if jac then jac:fill{ 24*x[1]^2, -1, -3*(1-x[1])^2, -1 } end
```

[7]This example is taken from the NLopt documentation.

```
    r:fill{ 8*x[1]^3-x[2], (1-x[1])^3-x[2] }
  end
  local x = vector{1.234, 5.678} -- start point
  local status, fmin, ncall = match {
    variables    = { rtol=1e-4,
                     { var='x[1]', min=-inf },
                     { var='x[2]', min=0 } },
    inequalities = { exec=testFuncLe, nequ=2, tol=1e-8 },
    objective    = { exec=testFuncFn, method='LD_MMA' },
    maxcall=100, info=2
  }
```

This example can also be solved with least squares methods, where the `LD_JACOBIAN` method finds the minimum in 8 iterations with a precision of $\pm 10^{-16}$, and the `LD_LMDIF` method finds the minimum in 10 iterations with a precision of $\pm 10^{-11}$.

Chapter 16. Correct

The `correct` command (i.e. orbit correction) provides a simple interface to compute the orbit steering correction and setup the kickers of the sequences from the analysis of their `track` and `twiss` mtables.

1 Command synopsis

Figure 16.1: Synopsis of the `correct` command with default setup.

```
mlst = correct {
  sequence=nil,    -- sequence(s) (required)
  range=nil,       -- sequence(s) range(s) (or sequence.range)
  title=nil,       -- title of mtable (default seq.name)
  model=nil,       -- mtable(s) with twiss functions (required)
  orbit=nil,       -- mtable(s) with measured orbit(s), or use model
  target=nil,      -- mtable(s) with target orbit(s), or zero orbit
  kind='ring',     -- 'line' or 'ring'
  plane='xy',      -- 'x', 'y' or 'xy'
  method='micado', -- 'LSQ', 'SVD' or 'MICADO'
  ncor=0,          -- number of correctors to consider by method, 0=all
  tol=1e-5,        -- rms tolerance on the orbit
  units=1,         -- units in [m] of the orbit
  corcnd=false,    -- precond of correctors using 'svdcnd' or 'pcacnd'
  corcut=0,        -- value to theshold singular values in precond
  cortol=0,        -- value to theshold correctors in svdcnd
  corset=true,     -- update correctors correction strengths
  monon=false,     -- fraction (0<?<=1) of randomly available monitors
  moncut=false,    -- cut monitors above moncut sigmas
  monerr=false,    -- 1:use mrex and mrey alignment errors of monitors
                   -- 2:use msex and msey scaling errors of monitors
  info=nil,        -- information level (output on terminal)
  debug=nil,       -- debug information level (output on terminal)
}
```

The `correct` command format is summarized in Figure 16.1, including the default setup of the attributes. The `correct` command supports the following attributes:

sequence The *sequence* (or a list of *sequence*) to analyze. (no default, required).
Example: `sequence = lhcb1`.

range A *range* (or a list of *range*) specifying the span of the sequence to analyze. If no range is provided, the command looks for a range attached to the sequence, i.e. the attribute `seq.range`. (default: `nil`).
Example: `range = "S.DS.L8.B1/E.DS.R8.B1"`.

title A *string* specifying the title of the *mtable*. If no title is provided, the command looks for the name of the sequence, i.e. the attribute `seq.name`. (default: `nil`).
Example: `title = "Correct orbit around IP5"`.

`model`	A *mtable* (or a list of *mtable*) providing `twiss`-like information, e.g. elements, orbits and optical functions, of the corresponding sequences. (no default, required). Example: `model = twmtbl`.
`orbit`	A *mtable* (or a list of *mtable*) providing `track`-like information, e.g. elements and measured orbits, of the corresponding sequences. If this attribute is `nil`, the model orbit is used. (default: `nil`). Example: `orbit = tkmtbl`.
`target`	A *mtable* (or a list of *mtable*) providing `track`-like information, e.g. elements and target orbits, of the corresponding sequences. If this attribute is `nil`, the design orbit is used. (default: `nil`). Example: `target = tgmtbl`.
`kind`	A *string* specifying the kind of correction to apply among `line` or `ring`. The kind `line` takes care of the causality between monitors, correctors and sequences directions, while the kind `ring` considers the system as periodic. (default: `'ring'`). Example: `kind = 'line'`.
`plane`	A *string* specifying the plane to correct among `x`, `y` and `xy`. (default: `'xy'`). Example: `plane = 'x'`.
`method`	A *string* specifying the method to use for correcting the orbit among `LSQ`, `SVD` or `micado`. These methods correspond to the solver used from the Matrix module to find the orbit correction, namely `solve`, `ssolve` or `nsolve`. (default: `'micado'`). Example: `method = 'svd'`.
`ncor`	A *number* specifying the number of correctors to consider with the method `micado`, zero meaning all available correctors. (default: `0`). Example: `ncor = 4`.
`tol`	A *number* specifying the rms tolerance on the residuals for the orbit correction. (default: `1e-5`). Example: `tol = 1e-6`.
`unit`	A *number* specifying the unit of the `orbit` and `target` coordinates. (default: `1` [m]). Example: `units = 1e-3` [m], i.e. [mm].
`corcnd`	A *logical* or a *string* specifying the method to use among `svdcnd` and `pcacnd` from the Matrix module for the preconditioning of the system. A `true` value corresponds to `svdcnd`. (default: `false`). Example: `corcnd = 'pcacnd'`.
`corcut`	A *number* specifying the thresholds for the singular values to pass to the `svdcnd` and `pcacnd` method for the preconditioning of the system. (default: `0`). Example: `cortol = 1e-6`.
`cortol`	A *number* specifying the thresholds for the correctors to pass to the `svdcnd` method for the preconditioning of the system. (default: `0`). Example: `cortol = 1e-8`.
`corset`	A *logical* specifying to update the correctors strengths for the corrected orbit. (default: `true`). Example: `corset = false`.

monon	A *number* specifying a fraction of available monitors selected from a uniform RNG. (default: `false`). Example: `monon = 0.8`, keep 80% of the monitors.
moncut	A *number* specifying a threshold in number of sigma to cut monitor considered as outliers. (default: `false`). Example: `moncut = 2`, cut monitors above 2σ.
monerr	A *number* in `0`..`3` specifying the type of monitor reading errors to consider: `1` use scaling errors `msex` and `msey`, `2` use alignment errors `mrex`, `mrey` and `dpsi`, `3` use both. (default: `false`). Example: `monerr = 3`.
info	A *number* specifying the information level to control the verbosity of the output on the console. (default: `nil`). Example: `info = 2`.
debug	A *number* specifying the debug level to perform extra assertions and to control the verbosity of the output on the console. (default: `nil`). Example: `debug = 2`.

The `correct` command returns the following object:

mlst	A *mtable* (or a list of *mtable*) corresponding to the TFS table of the `correct` command. It is a list when multiple sequences are corrected together.

2 Correct mtable

The `correct` command returns a *mtable* where the information described hereafter is the default list of fields written to the TFS files.[1]

The header of the *mtable* contains the fields in the default order:

name	The name of the command that created the *mtable*, e.g. `"correct"`.
type	The type of the *mtable*, i.e. `"correct"`.
title	The value of the command attribute `title`.
origin	The origin of the application that created the *mtable*, e.g. `"MAD 1.0.0 OSX 64"`.
date	The date of the creation of the *mtable*, e.g. `"27/05/20"`.
time	The time of the creation of the *mtable*, e.g. `"19:18:36"`.
refcol	The reference *column* for the *mtable* dictionnary, e.g. `"name"`.
range	The value of the command attribute `range`.[2]
__seq	The *sequence* from the command attribute `sequence`.[3]

The core of the *mtable* contains the columns in the default order:

[1]The output of mtable in TFS files can be fully customized by the user.
[2]This field is not saved in the TFS table by default.
[3]Fields and columns starting with two underscores are protected data and never saved to TFS files.

name	The name of the element.
kind	The kind of the element.
s	The s-position at the end of the element slice.
l	The length from the start of the element to the end of the element slice.
x_old	The local coordinate x at the s-position before correction.
y_old	The local coordinate y at the s-position before correction.
x	The predicted local coordinate x at the s-position after correction.
y	The predicted local coordinate y at the s-position after correction.
rx	The predicted local residual r_x at the s-position after correction.
ry	The predicted local residual r_y at the s-position after correction.
hkick_old	The local horizontal kick at the s-position before correction.
vkick_old	The local vertical kick at the s-position before correction.
hkick	The predicted local horizontal kick at the s-position after correction.
vkick	The predicted local vertical kick at the s-position after correction.
shared	A *logical* indicating if the element is shared with another sequence.
eidx	The index of the element in the sequence.

Note that `correct` does not take into account the particles and damaps `ids` present in the (augmented) `track` *mtable*, hence the provided tables should contain single particle or damap information.

3 Examples

TODO

Chapter 17. Emit

This command is not yet implemented in MAD. It will probably be implemented as a layer on top of the Twiss and Match commands.

Chapter 18. Plot

The `plot` command provides a simple interface to the Gnuplot application. The Gnuplot release 5.2 or higher must be installed and visible in the user `PATH` by MAD to be able to run this command.

1 Command synopsis

x x

Figure 18.1: Synopsis of the `plot` command with default setup.

```
cmd = plot {
  sid        = 1,   -- stream id 1 <= n <= 25 (Gnuplot instances)
  data       = nil, -- { x=tbl.x, y=vec } (precedence over table)
  table      = nil, -- mtable
  tablerange = nil, -- mtable range (default table.range)
  sequence   = nil, -- seq | { seq1, seq2, ...} | "keep"
  range      = nil, -- sequence range (default sequence.range)
  name       = nil, -- (default table.title)
  date       = nil, -- (default table.date)
  time       = nil, -- (default table.time)
  output     = nil, -- "filename" -> pdf | number -> wid
  scrdump    = nil, -- "filename"
  survey-attributes
  windows-attributes
  layout-attributes
  labels-attributes
  axis-attributes
  ranges-attributes
  data-attributes
  plots-attributes
  custom-attributes
  info=nil,  -- information level (output on terminal)
  debug=nil, -- debug information level (output on terminal)
}
```

The `plot` command format is summarized in Figure 18.1, including the default setup of the attributes. The `plot` command supports the following attributes:

`info` A *number* specifying the information level to control the verbosity of the output on the console. (default: `nil`).
Example: `info = 2`.

`debug` A *number* specifying the debug level to perform extra assertions and to control the verbosity of the output on the console. (default: `nil`).
Example: `debug = 2`.

The `plot` command returns itself.

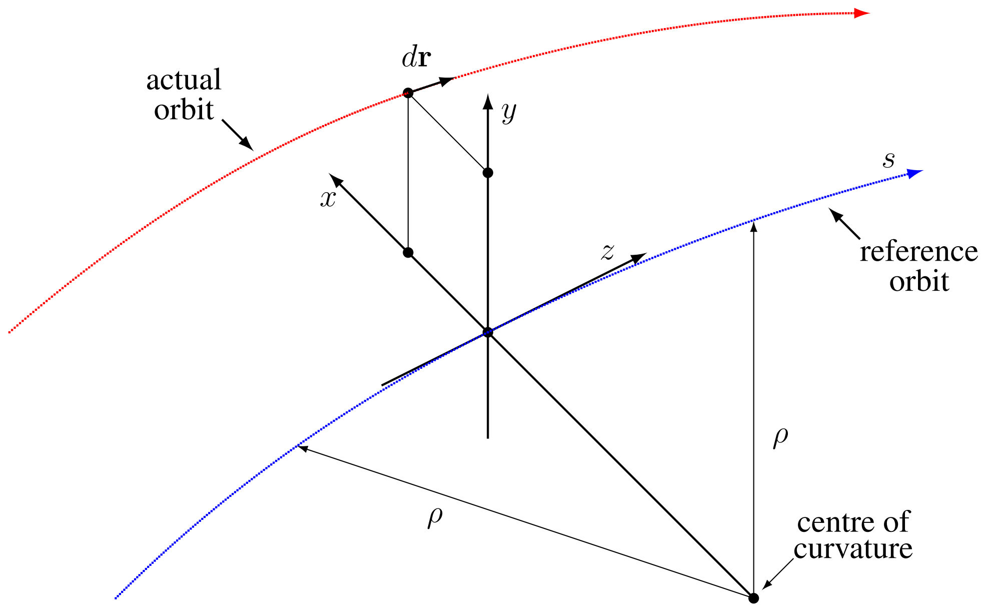
Part III

Physics

Chapter 19. Introduction

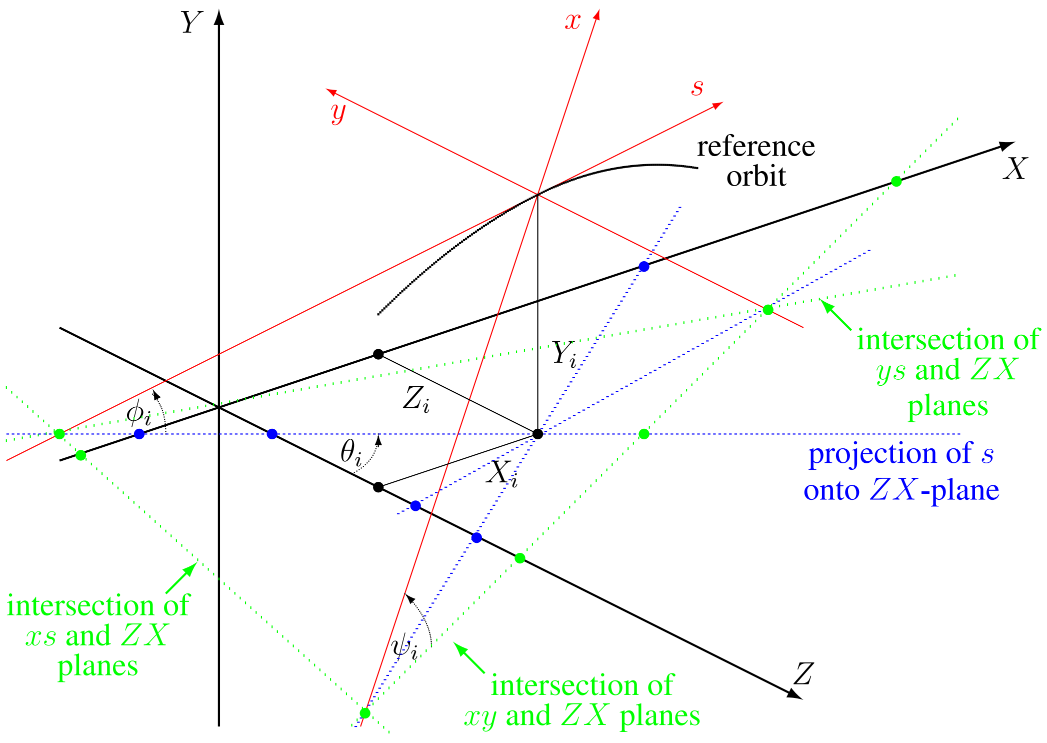
1 Local reference system

Figure 19.1: Local Reference System



2 Global reference system

Figure 19.2: Global Reference System showing the global Cartesian system (X, Y, Z) in black and the local reference system (x, y, s) in red after translation (X_i, Y_i, Z_i) and rotation $(\theta_i, \phi_i, \psi_i)$. The projections of the local reference system axes onto the horizontal ZX plane of the Cartesian system are figured with blue dashed lines. The intersections of planes ys, xy and xs of the local reference system with the horizontal ZX plane of the Cartesian system are figured in green dashed lines.



Y
x
s
y
reference orbit
X
Y_i
Z_i
ϕ_i
θ_i
X_i
intersection of ys and ZX planes
projection of s onto ZX-plane
intersection of xs and ZX planes
ψ_i
intersection of xy and ZX planes
Z

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